Supplementary Material for Optimal Redistribution in the Presence of Signaling

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1 Condition for full separation, $N > 2$ extended tax regime

Here we provide a heuristic derivation of the necessary and sufficient condition (39) for the optimality of a fully separating equilibrium. As a preliminary observation, notice that incentive-compatibility requires, $\forall i \in \{1, ..., N-1\}$, that $e^{i+1} \geq e^i$ and $c^{i+1} \geq c^i$; this implies that bunching can only involve adjacent types.[1] Then, suppose to start from an initial equilibrium where two adjacent types, i and $i+1$, are bunched together at a common bundle $(\widehat{y}, \widehat{c}, \widehat{e})$.[2] Consider first whether one can improve upon the initial equilibrium by achieving separation through a reform that raises the effort exerted by type-$i+1$ agents, without affecting their utility and without violating incentive-compatibility. For this purpose, suppose that the government replaces the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ with the bundle $(y^i, \widehat{c}, \widehat{e})$, where $y^i = \theta^i \widehat{e}$, and at the same time offers

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[1] The fact that $e^{i+1} \geq e^i$ and $c^{i+1} \geq c^i$ also implies that $y^{i+1} \geq y^i$.

[2] We will here consider the case where, at the initial equilibrium, no other type chooses the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$. Our argument can be easily generalized to the case where, at the initial equilibrium, more than two types are bunched together at the $(\widehat{y}, \widehat{c}, \widehat{e})$-bundle.

an additional bundle, which we will denote by $(y^{i+1}, c^{i+1}, e^{i+1})$, such that $e^{i+1} = \widehat{e} + \epsilon$, $c^{i+1} = \widehat{c} + \epsilon k^{i+1} g'(\widehat{e})$ and $y^{i+1} = (\widehat{e} + \epsilon)\,\theta^{i+1}$, with $\epsilon > 0$ and sufficiently small.[3] Clearly, type-i agents would not be tempted to choose the bundle $(y^{i+1}, c^{i+1}, e^{i+1})$ since their utility is strictly higher, by an amount equal to $(k^i - k^{i+1})\, g'(\widehat{e})\,\epsilon$, at the bundle $(y^i, \widehat{c}, \widehat{e})$ where their effort choice and well-being is the same as under the pre-reform equilibrium (where their bundle is $(\widehat{y}, \widehat{c}, \widehat{e})$). Type-$i+1$ agents would instead be just indifferent between the two bundles $(y^i, \widehat{c}, \widehat{e})$ and $(y^{i+1}, c^{i+1}, e^{i+1})$, which provide them the same utility that they enjoy under the pre-reform equilibrium (where their bundle is $(\widehat{y}, \widehat{c}, \widehat{e})$). Being indifferent between the two bundles $(y^i, \widehat{c}, \widehat{e})$ and $(y^{i+1}, c^{i+1}, e^{i+1})$, we can safely assume that all type-$i+1$ agents will choose the bundle $(y^{i+1}, c^{i+1}, e^{i+1})$. Notice however that, if at the initial equilibrium, type-$i+2$ agents were just indifferent between choosing the bundle $(y^{i+2}, c^{i+2}, e^{i+2})$, intended for them by the government, and the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$, after the reform they will be strictly better off (by an amount equal to $(k^{i+1} - k^{i+2})\, g'(\widehat{e})\,\epsilon$) choosing the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$. Thus, in order to preserve incentive-compatibility, the introduction of the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$ must be accompanied by an upward adjustment in c^{i+2} by $dc^{i+2} = (k^{i+1} - k^{i+2})\, g'(\widehat{e})\,\epsilon$. But once again, if at the initial allocation type-$i+3$ agents were just indifferent between choosing the bundle $(y^{i+3}, c^{i+3}, e^{i+3})$, intended for them by the government, and choosing the bundle $(y^{i+2}, c^{i+2}, e^{i+2})$, incentive-compatibility requires that c^{i+3} be adjusted upwards by the same amount as c^{i+2}. Replicating the same reasoning for all other types above $i+3$, one can conclude that offering the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$ requires to absorb a total amount of resources equal to $(k^{i+1} - k^{i+2})\, g'(\widehat{e}) \left(\sum_{j \geq i+2} \gamma^j\right) \epsilon$ in order to preserve incentive-compatibility. Thus, taking into account that total output in the economy would go up by $\gamma^{i+1}\theta^{i+1}\epsilon$ when type-$i+1$ agents switch from the initial bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ to the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$, the reform would allow the government to increase its net revenue by an amount equal to:

[3] Denote by $\overline{\theta}^{i,i+1}$ the average productivity of the agents that are bunched at the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$, i.e. $\overline{\theta}^{i,i+1} \equiv (\gamma^i\theta^i + \gamma^{i+1}\theta^{i+1}) \,/\, (\gamma^i + \gamma^{i+1})$. Notice that it must necessarily be that $\widehat{y} = \overline{\theta}^{i,i+1}\widehat{e}$. Thus, y^{i+1} can also be equivalently re-expressed as $y^{i+1} = \widehat{y} + \epsilon\theta^{i+1} + \left(\theta^{i+1} - \overline{\theta}^{i,i+1}\right)\widehat{e}$ and y^i can be equivalently re-expressed as $y^i = \widehat{y} - \left(\overline{\theta}^{i,i+1} - \theta^i\right)\widehat{e}$.

$$\left\{\gamma^{i+1}\theta^{i+1} - \left[\gamma^{i+1}k^{i+1} + \left(k^{i+1} - k^{i+2}\right)\left(\sum_{j\geq i+2}\gamma^{j}\right)\right] g'\left(\widehat{e}\right)\right\}\epsilon. \tag{1}$$

Provided that the quantity defined in (1) is positive, i.e. when

$$\frac{\gamma^{i+1}\theta^{i+1}}{\left[\gamma^{i+1}k^{i+1} + \left(k^{i+1} - k^{i+2}\right)\left(\sum_{j\geq i+2}\gamma^{j}\right)\right] g'\left(\widehat{e}\right)} > 1, \tag{2}$$

the pre-reform allocation, featuring bunching of type-i- and type-$i+1$ agents, is suboptimal. The reason is that, as we have shown above, one can implement a reform that, by separating the two types through an increase in the effort provided by type-$i+1$ agents, allows the government to obtain additional resources that can be used to raise social welfare.

Notice however that condition (2) is not a necessary condition to show that the initial allocation, featuring bunching of type-i- and type-$i+1$ agents, is suboptimal. In fact, another possibility to separate type-i- and type-$i+1$ agents is through a reform that achieves separation by lowering the effort exerted by type-i agents, without affecting their utility and without violating incentive-compatibility. For this purpose, consider the alternative reform where the government replaces the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ with the bundle $(y^{i+1}, \widehat{c}, \widehat{e})$, where $y^{i+1} = \theta^{i+1}\widehat{e}$, and at the same time offers an additional bundle, which we will denote by (y^{i}, c^{i}, e^{i}), such that $e^{i} = \widehat{e} - \epsilon$, $c^{i} = \widehat{c} - \epsilon k^{i} g'\left(\widehat{e}\right)$ and $y^{i} = \left(\widehat{e} - \epsilon\right)\theta^{i}$, with $\epsilon > 0$ and sufficiently small.[4] By construction, type-i agents would be just indifferent between the two bundles $(y^{i+1}, \widehat{c}, \widehat{e})$ and (y^{i}, c^{i}, e^{i}), which provide them the same utility that they enjoy under the pre-reform equilibrium (where their bundle is $(\widehat{y}, \widehat{c}, \widehat{e})$). Being indifferent between the two bundles $(y^{i+1}, \widehat{c}, \widehat{e})$ and (y^{i}, c^{i}, e^{i}), we can safely assume that all type-i agents will choose the latter bundle. Moreover, since type-$i+1$ agents are strictly better off at the initial bundle $(y^{i+1}, \widehat{c}, \widehat{e})$ than at the bundle (y^{i}, c^{i}, e^{i}), one can lower $\widehat{c}$ by $d\widehat{c} = \left(k^{i+1} - k^{i}\right) g'\left(\widehat{e}\right)\epsilon < 0$ without worrying that type-$i+1$ agents are induced to switch to the bundle (y^{i}, c^{i}, e^{i}). But once $\widehat{c}$ is lowered by $d\widehat{c} = \left(k^{i+1} - k^{i}\right) g'\left(\widehat{e}\right)\epsilon < 0$, one can afford to lower by the same amount, without

[4] Notice that y^{i} can also be equivalently re-expressed as $y^{i} = \widehat{y} - \epsilon\theta^{i} - \left(\overline{\theta}^{i,i+1} - \theta^{i}\right)\widehat{e}$ and y^{i+1} can be equivalently re-expressed as $y^{i+1} = \widehat{y} + \left(\theta^{i+1} - \overline{\theta}^{i,i+1}\right)\widehat{e}$.

violating incentive-compatibility, also the consumption available to agents choosing the bundles $(y^{i+2}, c^{i+2}, e^{i+2})$, $(y^{i+3}, c^{i+3}, e^{i+3})$, and so on. Thus, taking into account that total output in the economy would go down by $\gamma^i \theta^i \epsilon$ when type-i agents switch from the initial bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ to the new bundle (y^i, c^i, e^i), the reform would allow the government to increase its net revenue by an amount equal to:

$$\left\{ -\gamma^i \theta^i + \left[\gamma^i k^i + \left(k^i - k^{i+1} \right) \left(\sum_{j \geq i+1} \gamma^j \right) \right] g' \left(\widehat{e} \right) \right\} \epsilon. \tag{3}$$

Provided that the quantity defined in (1) is positive, i.e. when

$$\frac{\gamma^i \theta^i}{\left[\gamma^i k^i + \left(k^i - k^{i+1} \right) \left(\sum_{j \geq i+1} \gamma^j \right) \right] g' \left(\widehat{e} \right)} < 1, \tag{4}$$

the pre-reform allocation, featuring bunching of type-i- and type-$i+1$ agents, is suboptimal. The reason is that one can implement a reform that, by separating the two types through an reduction in the effort provided by type-i agents, allows the government to obtain additional resources that can be used to raise social welfare.

Thus, putting together conditions (2) and (4), one can conclude that a necessary and sufficient condition for the suboptimality of the pre-reform allocation, featuring bunching of type-i- and type-$i+1$ agents, is

$$\frac{\gamma^{i+1} \theta^{i+1}}{\left[\gamma^{i+1} k^{i+1} + \left(k^{i+1} - k^{i+2} \right) \left(\sum_{j \geq i+2} \gamma^j \right) \right] g' \left(\widehat{e} \right)} > \frac{\gamma^i \theta^i}{\left[\gamma^i k^i + \left(k^i - k^{i+1} \right) \left(\sum_{j \geq i+1} \gamma^j \right) \right] g' \left(\widehat{e} \right)}, \tag{5}$$

or, equivalently, condition (39) in the main document.

2 Optimal distortions in the two-dimensional case

Consider the problem solved by a government under an extended tax regime and denote μ the Lagrange multiplier attached to the constraint (63), by λ^2 the multiplier attached to the constraint (64) and by λ^1 the multiplier attached to the constraint (65). The first order conditions for y^1, e_s^1, c^1, y^2, e_s^2, c^2 are respectively given by:

$$\left(1+\lambda^1\right)\frac{\partial R^1\left(e_s^1,e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1,e_s^1,\theta^1\right)}\frac{\partial e_q^{1*}}{\partial y^1}=\lambda^2\frac{\partial R^2\left(e_s^1,\hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1,e_s^1,\overline{\theta}\right)}\frac{\partial \hat{e}_q^2}{\partial y^1}+\mu\gamma^1, \tag{6}$$

$$\begin{aligned}&\left(1+\lambda^1\right)\left[\frac{\partial R^1\left(e_s^1,e_q^{1*}\right)}{\partial e_s^1}+\frac{\partial R^1\left(e_s^1,e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1,e_s^1,\theta^1\right)}\frac{\partial e_q^{1*}}{\partial e_s^1}\right]\\ =\;&\lambda^2\left[\frac{\partial R^2\left(e_s^1,\hat{e}_q^2\right)}{\partial e_s^1}+\frac{\partial R^2\left(e_s^1,\hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1,e_s^1,\overline{\theta}\right)}\frac{\partial \hat{e}_q^2}{\partial e_s^1}\right],\end{aligned} \tag{7}$$

$$1+\lambda^1=\lambda^2+\mu\gamma^1, \tag{8}$$

$$\lambda^2\frac{\partial R^2\left(e_s^2,e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2,e_s^2,\theta^2\right)}\frac{\partial e_q^{2*}}{\partial y^2}=\lambda^1\frac{\partial R^1\left(e_s^2,e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2,e_s^2,\theta^2\right)}\frac{\partial e_q^{2*}}{\partial y^2}+\mu\gamma^2, \tag{9}$$

$$\begin{aligned}&\lambda^2\left[\frac{\partial R^2\left(e_s^2,e_q^{2*}\right)}{\partial e_s^2}+\frac{\partial R^2\left(e_s^2,e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2,e_s^2,\theta^2\right)}\frac{\partial e_q^{2*}}{\partial e_s^2}\right]\\ =\;&\lambda^1\left[\frac{\partial R^1\left(e_s^2,e_q^{2*}\right)}{\partial e_s^2}+\frac{\partial R^1\left(e_s^2,e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2,e_s^2,\theta^2\right)}\frac{\partial e_q^{2*}}{\partial e_s^2}\right],\end{aligned} \tag{10}$$

$$\lambda^2=\lambda^1+\mu\gamma^2. \tag{11}$$

Dividing (7) by (8), and multiplying both sides by the RHS of (8) gives:

$$\begin{aligned}&\left[\frac{\partial R^1\left(e_s^1,e_q^{1*}\right)}{\partial e_s^1}+\frac{\partial R^1\left(e_s^1,e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1,e_s^1,\theta^1\right)}\frac{\partial e_q^{1*}}{\partial e_s^1}\right]\left(\lambda^2+\mu\gamma^1\right)\\ =\;&\lambda^2\left[\frac{\partial R^2\left(e_s^1,\hat{e}_q^2\right)}{\partial e_s^1}+\frac{\partial R^2\left(e_s^1,\hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1,e_s^1,\overline{\theta}\right)}\frac{\partial \hat{e}_q^2}{\partial e_s^1}\right],\end{aligned} \tag{12}$$

which can be equivalently rewritten as

$$\left[p_s+p_q^1\frac{\partial e_q^{1*}}{\partial e_s^1}\right]\left(\lambda^2+\mu\gamma^1\right)=\lambda^2\left[p_s+p_q^2\frac{\partial \hat{e}_q^2}{\partial e_s^1}\right],$$

from which one obtains

$$p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1} = \frac{\lambda^2}{\mu\gamma^1}\left[\left(p_s + p_q^2 \frac{\partial \hat{e}_q^2}{\partial e_s^1}\right) - \left(p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1}\right)\right], \tag{13}$$

and therefore, simplifying terms,

$$p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1} = \frac{\lambda^2}{\mu\gamma^1}\left[p_q^2 \frac{\partial \hat{e}_q^2}{\partial e_s^1} - p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1}\right]. \tag{14}$$

Notice that we have

$$\frac{\partial e_q^{1*}}{\partial e_s^1} = -\frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} \text{ and } \frac{\partial \hat{e}_q^2}{\partial e_s^1} = -\frac{\partial h\left(e_s^1, \hat{e}_q^2\right)/\partial e_s}{\partial h\left(e_s^1, \hat{e}_q^2\right)/\partial e_q}. \tag{15}$$

Moreover, since $e_q^{1*} = e_q^{1*}\left(y^1, e_s^1, \theta^1\right)$ and $\hat{e}_q^2 = \left(y^1, e_s^1, \overline{\theta}\right)$, it follows that $e_q^{1*} > \hat{e}_q^2 > 0$ and therefore $\frac{\partial e_q^{1*}}{\partial e_s^1} < \frac{\partial \hat{e}_q^2}{\partial e_s^1} < 0$. Thus, the RHS of (14) is positive (given that our max-min objective implies that $\lambda^2 > 0$). Since $p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1} = p_s - p_q^1 \frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q}$, we can conclude that at an optimal separating equilibrium

$$p_s - p_q^1 \frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} > 0. \tag{16}$$

Dividing (10) by (11), and multiplying both sides by the RHS of (11) gives:

$$\begin{aligned}
&\left[\frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_s^2} + \frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial e_s^2}\right]\left(\lambda^1 + \mu\gamma^2\right) \\
=\ &\lambda^1 \left[\frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_s^2} + \frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial e_s^2}\right],
\end{aligned} \tag{17}$$

which can be equivalently rewritten as

$$\left[p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2}\right]\left(\lambda^1 + \mu\gamma^2\right) = \lambda^1 \left[p_s + p_q^1 \frac{\partial e_q^{2*}}{\partial e_s^2}\right], \tag{18}$$

from which one obtains

$$p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2} = \frac{\lambda^1}{\mu\gamma^2}\left[\left(p_s + p_q^1 \frac{\partial e_q^{2*}}{\partial e_s^2}\right) - \left(p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2}\right)\right], \tag{19}$$

and therefore, simplifying terms:

$$p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2} = \frac{\lambda^1}{\mu\gamma^2} \frac{\partial e_q^{2*}}{\partial e_s^2} \left(p_q^1 - p_q^2\right). \tag{20}$$

Since we have that $p_q^1 - p_q^2 > 0$ and

$$\frac{\partial e_q^{2*}}{\partial e_s^2} = -\frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} < 0, \tag{21}$$

it follows that the RHS of (20) is either negative (when $\lambda^1 > 0$) or zero (when $\lambda^1 = 0$). Since $p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2} = p_s - p_q^2 \frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q}$, we can conclude that at an optimal separating equilibrium

$$p_s - p_q^2 \frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} \leq 0. \tag{22}$$

Dividing (6) by (8), and multiplying both sides by the RHS of (8) gives:

$$\frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1, e_s^1, \theta^1\right)} \frac{\partial e_q^{1*}}{\partial y^1} \left(\lambda^2 + \mu\gamma^1\right) = \lambda^2 \frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1, e_s^1, \overline{\theta}\right)} \frac{\partial \hat{e}_q^2}{\partial y^1} + \mu\gamma^1, \tag{23}$$

which can be equivalently rewritten as

$$p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} \left(\lambda^2 + \mu\gamma^1\right) = \lambda^2 p_q^2 \frac{\partial \hat{e}_q^2}{\partial y^1} + \mu\gamma^1, \tag{24}$$

from which one obtains

$$1 - p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} = \frac{\lambda^2}{\mu\gamma^1} \left[p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} - p_q^2 \frac{\partial \hat{e}_q^2}{\partial y^1}\right]. \tag{25}$$

Since we have that

$$\frac{\partial e_q^{1*}}{\partial y^1} = \frac{1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} \quad \text{and} \quad \frac{\partial \hat{e}_q^2}{\partial y^1} = \frac{1}{\overline{\theta} \partial h\left(e_s^1, \hat{e}_q^2\right)/\partial e_q}, \tag{26}$$

it follows that $\frac{\partial e_q^{1*}}{\partial y^1} > \frac{\partial \hat{e}_q^2}{\partial y^1}$ (taking into account that $\theta^1 < \overline{\theta}$ and $e_q^{1*} > \hat{e}_q^2 > 0$). Thus, the RHS of (25) is strictly positive (given that our max-min objective implies that $\lambda^2 > 0$). Since $1 - p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} = 1 - \frac{p_q^1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q}$, it follows that at an optimal separating equilibrium

$$1 - \frac{p_q^1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} > 0. \tag{27}$$

Dividing (9) by (11), and multiplying both sides by the RHS of (11) gives:

$$\frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial y^2}\left(\lambda^1 + \mu\gamma^2\right) = \lambda^1 \frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial y^2} + \mu\gamma^2, \tag{28}$$

which can be equivalently rewritten as

$$p_q^2 \frac{\partial e_q^{2*}}{\partial y^2}\left(\lambda^1 + \mu\gamma^2\right) = \lambda^1 p_q^1 \frac{\partial e_q^{2*}}{\partial y^2} + \mu\gamma^2, \tag{29}$$

from which one obtains

$$1 - p_q^2 \frac{\partial e_q^{2*}}{\partial y^2} = \frac{\lambda^1}{\mu\gamma^2} \frac{\partial e_q^{2*}}{\partial y^2}\left(p_q^2 - p_q^1\right). \tag{30}$$

Given that $p_q^2 - p_q^1 < 0$, it follows that the RHS of (30) is either negative (when $\lambda^1 > 0$) or zero (when $\lambda^1 = 0$). Since $1 - p_q^2 \frac{\partial e_q^{2*}}{\partial y^2} = 1 - \frac{p_q^2}{\theta^2 \partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q}$, it follows that at an optimal separating equilibrium

$$1 - \frac{p_q^2}{\theta^2 \partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} \leq 0. \tag{31}$$