Optimal (P)redistribution and Education Signaling*

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May 27, 2022

Abstract

We study optimal redistribution when workers' innate productive abilities are neither observed by the government nor by prospective employers. Workers signal their productivity through their educational attainment reflecting both quantity (e.g., years of schooling) and quality dimensions (e.g., reputation of certifying institute). The dual role of income taxation in redistributing income and affecting signaling incentives is analyzed. We also study the role of extended tax systems that supplement income taxation with taxes/subsidies on education. Our analysis highlights when it is feasible and socially desirable for the government to achieve predistribution, namely, redistribution through wage compression.

Keywords: nonlinear taxation, education, asymmetric information, human capital

JEL classification: D82, H21, H52, J31

*Spencer Bastani gratefully acknowledges financial support from Jan Wallanders och Tom Hedelius stiftelse (grant P2018-162).

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1 Introduction

In the canonical framework of optimal income taxation, pioneered by Mirrlees (1971), asymmetric information between the government and private agents is the major constraint on public policy. The government desires to redistribute between individuals on the basis of their innate productive abilities, but as these abilities cannot be observed for tax purposes, it has to rely on the taxation of income and other observable quantities, serving as proxies for the (unobserved) abilities. This results in a second-best problem where incentive-compatibility (hereafter, IC) considerations warrant the introduction of distortions, typically taking the form of positive marginal tax rates (with extensive margin choices in place, such as migration and labor market participation, negative marginal tax rates can be desirable).

Since the seminal contributions by Spence (1973) and Akerlof (1976), economists have recognized that asymmetric information in the labor market has a major impact on the nature of interactions between employees and firms, and can be an important source of inefficiency in market outcomes. It implies that employers cannot observe the productivity of workers, and hence, even in a competitive labor market, workers are not necessarily compensated according to their marginal product. Instead, the wage distribution becomes endogenous, and is affected by the screening/signaling possibilities available to employers/workers.

Two recent papers have revisited the Mirrlees setup and extended it by introducing a second source of asymmetric information between workers and employers. They emphasize how firms screen between workers on the basis of their choice of working hours via inducing high-skill workers to work more than the efficient amount in exchange for a higher compensation (rat-race). Stantcheva (2014) studies how this adverse selection aspect of the labor market affects the design of optimal income taxation, showing that firms' use of work hours and pay as screening tools can help the government to redistribute as it may constrain the adverse reactions of high types to progressive taxation. Bastani et al. (2015) use a similar screening setup but discuss how the government might choose to use its available policy instruments to affect the wage distribution by implementing bunching or pooling of types.[1] Intuitively, if the government makes it harder for workers and firms to exchange information in the labor market, this can serve redistributive purposes as two workers with different productivity levels may end up with the same market remuneration. Notably, this happens even when the production technology is linear and skill types are perfect substitutes, as in Mirrlees (1971).[2]

The purpose of the current paper is to make progress in developing a framework to analyze optimal redistributive policy in the presence of (impure) signaling, allowing for the possibility of redistribution through wage compression. In line with the optimal income tax literature, we

[1] Stantcheva (2014) focuses on the Miyazaki-Wilson-Spence equilibrium whereas Bastani et al. (2015) use the Rotschild-Stiglitz equilibrium concept.

[2] The mechanism is therefore different from the one emphasized by papers studying optimal income taxation in a general equilibrium context where redistribution through the wage channel occurs due to sector re-allocations of labor (see e.g., Stiglitz 1982, Rothschild and Scheuer 2013 and Sachs et al. 2020).

assume that workers differ in their innate productive abilities which are unobservable to the government. However, in contrast to the vast majority of papers in the optimal income tax literature, but in similarity to the two papers discussed above, we assume that these abilities are also unobservable to prospective employers. The distinguishing feature of our setup is that workers must signal their type to firms through costly effort choices.[3] Moreover, we allow the information transmission between workers and firms to occur along more than one dimension, and we analyze extended tax systems that combine income taxation with taxes on the signals. As we show, these aspects are essential to understand how optimal redistributive policy should be designed in economies with two layers of asymmetric information. In particular, they determine the feasibility and social desirability of redistribution through wage compression.

The income tax system is an indirect instrument to tax the signaling activities that take place in the labor market. However, in many relevant circumstances, direct instruments affecting the transmission of information between workers and their actual (or prospective) employers can be implemented as well, namely, instruments that tax the signals themselves.[4] These direct instruments may take the form of education mandates (specifying a minimum level of education), overtime regulation or worker monitoring technologies. In the standard Mirrlees setup, the possibility to observe labor effort (alongside income) would essentially allow the government to tax agents according to their innate abilities. With a second source of asymmetric information, however, employers cannot distinguish between workers with different abilities, and need to rely on the observed signals strategically chosen by workers. This implies that a first-best optimum cannot be achieved, even when direct instruments are available.

Our vehicle of analysis is a model that captures the equity-efficiency trade-off in a similar vein as the two-type Stiglitz (1982) version of the Mirrlees (1971) optimal income tax model. We characterize optimal nonlinear tax policy by focusing on feasible and incentive-compatible allocations, invoking the revelation principle, and solving for the optimal direct revelation mechanism. To illustrate our framework, we begin considering signaling in one dimension. We then consider the more realistic case in which employers, while still being uninformed about workers' productivities, possess better information than the government. We do so by assuming that workers' signaling has two dimensions: quantity (e.g., years of schooling) which is universally observable (that is, both by the government and the employers) and quality (such as the difficulty or intensity of a particular education track) that is exclusively observed by the employers. To render signaling feasible, we assume that workers not only differ in their innate productive ability, but also in their costs of signaling (e.g., in the costs of obtaining education). A natural interpretation of the signals in our model is that they represent components of education

[3]By focusing on signaling, our study is related to Craig (2021) who studies the design of optimal income taxation in a different setup where employers perform Bayesian inference regarding workers' productivity. In his setting, the equilibrium wage is a weighted average of the worker's own productivity and the productivity of other similar workers.

[4]This topic has received surprisingly little attention in the optimal income tax literature. The only previous contribution we are aware of that explicitly discusses the taxation of signals is Andersson (1996). Other papers discussing signaling in the context of taxation are Spence (1974) and Manoli (2006)

effort. In line with this interpretation, the signals in our model are not pure waste, but realistically also enhance human capital.[5]

Our main results can be summarized as follows. With signaling confined to a single dimension, and when only income can be taxed, the social optimum is given by either a separating allocation, where redistribution takes place *ex post* by compressing the after-tax income distribution, or a pooling allocation, where taxation exerts predistributive effects (see e.g., Bozio et al. 2020), i.e. it redistributes *ex ante* (through the wage channel) by compressing the pre-tax wage distribution. However, when the income tax is supplemented by a direct tax on the signal, the optimum is always given by a separating allocation. The reason is that a direct tax on the signal enables to fully eliminate the information rents associated with differences in productivities, which eliminates the case for redistribution through wage compression (pooling). Due to the presence of a second layer of asymmetric information, however, a first-best is not necessarily attainable, as there remain information rents associated with the differences in the cost of signaling. The presence of binding IC constraints implies that it is optimal for the government to impose distortions on the labor/signaling choices of individuals, distortions that differ from the standard ones typically emphasized in optimal tax models.

In the presence of bi-dimensional signaling, and when only income can be taxed, we show that a pooling equilibrium does not exist, rendering redistribution through the wage channel infeasible. However, by supplementing the income tax with a direct tax levied on the signal observable by the government, the possibility of implementing a pooling equilibrium (and thereby achieve redistribution through the wage channel) is restored. In contrast to the model with one signal (where taxing the signal always renders the pooling equilibrium sub-optimal), levying a tax on one of the two signals available to agents does not rule out the social desirability of a pooling equilibrium. The reason is that the information rents associated with differences in productivities are not fully eliminated by taxing only one of the signals available to workers. We illustrate the nature of the social optimum and the associated welfare gains depending on the parameters of the economy through several numerical examples. These examples highlight both the role of the difference in the inherent productivities of agents, as well as the differences in the costs of signaling.

Government provision and/or subsidization of education is often justified on efficiency grounds (e.g., alleviating credit constraints and internalizing externalities), but also serves to promote redistributive goals, as a supplement to income taxation.[6] Interpreting signaling in

[5]Our paper thus relates to papers that study the design of optimal income taxation in the presence of human capital investments and learning-by-doing, see e.g., Stantcheva (2017).

[6]Education subsidies feature frequently in the optimal tax literature, typically serving a dual role of i) mitigating/offsetting the dis-incentivizing effect of income taxation on human capital acquisition, and ii) enhancing redistribution (see, inter-alia, Ulph 1977, Tuomala 1986, Boadway and Marchand 1995, Brett and Weymark 2003, Bovenberg and Jacobs 2005, Maldonado 2008). Other studies have highlighted the potential desirability of education taxes. For example, Blumkin and Sadka (2008) argue that a positive correlation between observed educational attainment and unobserved innate productive ability provides a rationale for education taxes. More recently, Findeisen and Sachs (2016) analyze income-contingent student loans and find that it might be optimal for the government to enforce very rich individuals to pay back more than the capitalized loan value, effectively

the context of educational attainment, our analysis provides a novel normative justification for education taxes/subsidies, or mandates, as useful complements to income taxation. When signaling is confined to one dimension, our results alludes to the potential redistributive role played by setting an education mandate, in the form of compulsory schooling, as a means to enhance the target-efficiency of the tax and transfer system. The case featuring signaling along two dimensions suggests that (means-tested) education taxes/subsidies can be desirable when implementing a separating allocation: for instance, if the government observes the signal in which low-skilled workers have a comparative advantage, education subsidies should be offered to low-income workers, whereas it might be desirable to impose education taxes on high-income workers. Both instruments serve the role of alleviating the binding IC constrain(s), thereby enhancing the extent of redistribution that can be achieved through income taxation. When signaling occurs along two dimensions and a pooling allocation constitutes the social optimum, an education mandate is instead desirable as a means to prevent cream skimming and support cross-subsidization via the wage channel.

The paper is organized as follows. In Section 2 we analyze optimal redistribution in the presence of signaling in one dimension. Section 3 considers a model with signaling in two dimensions, capturing the possibility for firms to have an informational advantage vis-a-vis the government. In Section 4 we make a specific functional form assumption which allows us to present further analytical results for the bi-dimensional signaling case, and we also provide an illustrative numerical example. Section 5 concludes.

2 A model with signaling in one dimension

Consider an economy comprised of two types of workers: a low-skilled worker, denoted by $i = 1$, and a high-skilled worker denoted by $i = 2$, who differ in their innate ability.[7] The fraction of type-i workers in the population (normalized to a unit measure, with no loss of generality) is denoted by $0 < \gamma^i < 1$. The labor market is competitive, but the innate ability of a worker is assumed to be private information unavailable to the firm. Thus, we deviate from the standard Mirrleesian setup by considering two layers of asymmetric information, one between the government and private agents, and one between workers and firms.[8] Workers exert costly effort denoted by e^i which serves a dual purpose: (i) increasing the productivity of the worker, and, (ii) signaling innate ability. Our model is general, but for concreteness, we focus on educational attainment and make the natural interpretation of e^i as education effort prior to entering the labor market. In line with this interpretation, workers are first-movers in the interaction with firms. Without being excessively unrealistic and in order to simplify the

implying a tax on education.

[7] We briefly discuss how our results generalize to more than two types when we present our main result in this section, stated in Proposition 3.

[8] Other papers that have considered two layers of asymmetric information are Stantcheva (2014), Bastani et al. (2015, 2019), and Craig (2021).

exposition and render the setup more tractable, we assume that labor supply is inelastic and normalized to one unit of time. The output of worker i per unit of time is given by $z^i = e^i\theta^i$ where θ^i denotes the innate ability of type i, with $\theta^2 > \theta^1 > 0$. The utility of a type-i worker is given by:

$$u^i(c, e) = c - g^i(e), \tag{1}$$

where c denotes consumption and g^i denotes the cost of exerting effort and is assumed to be a strictly increasing and convex function.[9] Moreover, the cost of acquiring effort (both marginal and total) decreases with respect to the skill level, i.e., $g^2(e) < g^1(e)$ and $\partial g^2(e)/\partial e < \partial g^1(e)/\partial e$. These are standard properties in the signaling literature and ensure that the single-crossing property, that is essential for the feasibility of a separating (fully revealing) equilibrium, is satisfied. To simplify, we let $g^2(e) \equiv g(e)$ and $g^1(e) \equiv kg(e)$ where we assume $k > 1$ so that low-skilled agents incur a higher effort cost, for any given effort level. The firms observe e^i, which serves as a signal for the unobserved innate productive ability of the worker.

2.1 Laissez-faire equilibrium

Given that we are mainly interested in the equilibrium with taxes/transfers in place, we will here only provide a brief treatment of the laissez-faire equilibrium. We define a Bayesian Nash Equilibrium for the signaling game in the absence of government intervention as follows. The signaling game is comprised of two stages. In the first stage, workers choose their level of effort e^i, $i = 1, 2$. In the second stage, each firm offers a labor contract which specifies the income level (which is also the consumption level with no taxes or transfers in place) as a function of the observed effort levels, namely, $y^i(e^i)$. Based on the observable effort levels, firms form their beliefs with respect to the workers' types. In equilibrium, choices are consistent in the sense that firms maximize their expected profits by choosing labor contracts, given their beliefs; and, workers maximize their utility (by choosing their effort level) given the labor contracts offered by firms.[10] Assuming a perfectly competitive labor market with free entry of firms implies that rents are fully dissipated, hence, $y^i = e^i\theta^i$ and the compensation of all agents is equal to their production.

In the laissez-faire, a separating equilibrium exists provided the fraction of low-types is sufficiently high (see e.g., Riley 2001), whereas a pooling equilibrium is not stable due to a violation of the Intuitive Criterion (Cho and Kreps 1987). Due to the asymmetric information between firms and workers (with respect to innate ability) a laissez-faire separating equilibrium

[9]We use the quasi-linear specification for tractability. The qualitative features of our results could be obtained under more general utility specifications.

[10]For a more formal treatment see, e.g., Fudenberg and Tirole (1991).

has to satisfy IC constraints. In particular, the effort e^{1*} chosen by type-1 agents is:

$$e^{1*} = \underset{e^1}{\operatorname{argmax}} \left\{ e^1\theta^1 - kg(e^1) \right\}, \tag{2}$$

whereas the effort chosen by type-2 agents, e^{2*}, is given by

$$e^{2*} = \underset{e^2}{\operatorname{argmax}} \left\{ e^2\theta^2 - g(e^2) \right\} \tag{3}$$

$$\text{s.t.} \quad e^{1*}\theta^1 - kg(e^{1*}) \geq e^2\theta^2 - kg(e^2). \tag{4}$$

According to problem (2), type-1 agents exert their efficient effort level, which is implicitly characterized by the first order condition $g'(e^1) = \theta^1/k$. Type-2 agents, instead, have to exert an effort level that credibly signals their higher productivity to the firms. This requires that e^{2*} must satisfy the IC constraint (4).[11] The reason for this constraint is that, if the difference in the cost of acquiring the signal is not too large (i.e., if k is not too large), low-skilled agents may find it desirable to replicate the effort of high-skilled agents in order to receive a higher remuneration (per unit of effort). Therefore, an adverse selection problem generally arises, implying that type-2 workers may be induced to choose an inefficiently high level of effort. As we will see, the possibility of a binding upward constraint carries over to the optimal income tax problem, which implies the possibility of negative optimal marginal tax rates at the top.

2.2 The income tax regime

Suppose that the government is seeking to design a nonlinear income tax system that serves redistributive purposes. In particular, assume that the government is invoking a max-min social welfare function and is hence interested in maximizing the utility of the least well-off workers (type-1) subject to a balanced budget constraint (we assume no exogenous revenue needs with no loss of generality).[12] The government can choose any non-linear income tax system defined by a set of pre-tax/post-tax income bundles denoted by (y^i, c^i) where the total tax (or transfer if negative) is defined by $t^i \equiv y^i - c^i$. Doing so will lead to an equilibrium given the game played by firms and workers, which we call a *tax equilibrium*. Our exposition draws upon the equivalence between the set of tax equilibria and the set of allocations solving certain constrained optimization problems that will be outlined below.

For later purposes, define the wage rate earned by a given individual as the ratio between his/her pre-tax income and his/her effort. Due to the second layer of asymmetric information between the firms and the workers, the equilibrium wage distribution is affected by the tax-and-

[11] The set of contracts $\left\{ \left(e^{1*}, e^{1*}\theta^1\right), \left(e^{2*}, e^{2*}\theta^2\right) \right\}$ represents the unique Nash equilibrium pair of contracts which satisfies the intuitive criterion proposed by Cho and Kreps (1987). We further discuss this commonly applied refinement condition and its policy implications in subsection 2.2.1 below.

[12] The qualitative features of our analysis would not change if we would consider a more general egalitarian social welfare function.

transfer system implemented by the government. This enables the government to use the wage channel as a supplementary tool to the standard income channel to attain enhanced redistribution. In particular, the government can, by a proper choice of the tax schedule, implement either a separating or a pooling equilibrium.

An important observation is that, unlike the standard case with a single layer of asymmetric information (between the government and agents), the pooling tax equilibrium is *not* a special case of the separating tax equilibrium, and hence each type of tax equilibrium must be analyzed separately. The reason being, that in a separating equilibrium, each individual is remunerated exactly according to his/her ability. A necessary condition for this to happen, is that there is variation in income, so that high-skilled agents can distinguish themselves from their lower-skilled counterparts through signaling.

2.2.1 A separating equilibrium

If an optimal tax system leads to a separating equilibrium, then it must solve the following maximization problem, henceforth problem $\mathcal{P}1$:

Problem $\mathcal{P}1$

$$\max_{\{y^1,c^1,y^2,c^2\}} \quad c^1 - kg(e^1)$$

subject to:

$$y^i = e^i\theta^i, \quad i = 1, 2 \tag{5}$$

$$c^1 - kg(e^1) \geq c^2 - kg(e^2), \tag{6}$$

$$c^2 - g(e^2) \geq c^1 - g(e^1), \tag{7}$$

$$\sum_i \gamma^i(y^i - c^i) = 0. \tag{8}$$

Conditions (6)-(7) are the standard IC constraints, which ensure no mimicking. Each type weakly prefers his bundle over the one associated with his counterpart. Condition (8) states the government revenue constraint which ensures a balanced budget. Notice that redistribution from high- to low-skilled agents invites mimicking by high-skilled agents, who might be tempted to earn the income of low-skilled agents in order to qualify for a lower tax burden. At the same time, low-skilled agents might have an incentive to mimic high-skilled agents in order to qualify for a higher wage rate (i.e., for a higher remuneration per unit of effort), even though they would be subject to higher income taxation. Notice that the IC constraints (6)-(7) cannot both be binding at the same time by virtue of the fact that preferences satisfy the single-crossing property in the (c, e)-space (reflected by the fact that $k > 1$). For any egalitarian welfare function the one which is binding is hence the downward constraint. This clearly applies to the max-min case studied here. Finally, it is worth observing that in the solution to the above problem, high-skilled workers derive no information rent from their higher productivity.

A limitation of the above statement of a separating equilibrium in the presence of an income tax is that it does not consider the possibility for off-equilibrium deviations. In order to accommodate these potential stability threats, we will rely on the extension, provided by Grossman and Perry (1986), of the intuitive criterion proposed by Cho and Kreps (1987), which imposes some natural restrictions on out-of-equilibrium beliefs (the joint deviations of both types).[13]

Let T denote the set of all types (in our case $T = \{1, 2\}$) and let $S \subseteq T$ denote a subset of T. Suppose all types in the subset S deviate by choosing an effort level, $\hat{e}$, different than those specified (for their respective types) in equilibrium. Three questions need to be asked:

(a) Assuming that all firms believe that the deviating types come from the subset S (and accordingly update their beliefs in a Bayesian fashion), what would be the maximum (gross) income that firms could offer? Denote this income by $\hat{y}$. A deviation to $\hat{e}$ thus defines a new contract (which is not part of the presumably stable equilibrium) given by the triplet $(\hat{e}, \hat{y}, \hat{c})$, where $\hat{c}$ denotes the net income associated with $\hat{y}$.

(b) Would any type $s \in S$ be strictly better off with $(\hat{e}, \hat{y}, \hat{c})$ than with his equilibrium bundle?

(c) Would any type $s \notin S$ be weakly better off with the equilibrium bundle than with the contract $(\hat{e}, \hat{y}, \hat{c})$?

If the answer to both (b) and (c) is yes, then any type in S finds it profitable to deviate, as he/she is able to credibly distinguish himself from types that are not in S and thereby become strictly better off.[14]

Turning back to the separating equilibrium defined above, we acknowledge that the only possible deviations in the presence of the income tax are those associated with the income levels y^1 and y^2.[15] Consider a deviation to the following bundle: $(\hat{e}, y^1, c^1)$, where $\hat{e} = \frac{y^1}{\sum_i \gamma^i \theta^i}$. As $\theta^2 > \theta^1$, we have that $\hat{e} < e^1 = \frac{y^1}{\theta^1}$. It thus follows that type-1 strictly prefers the bundle $(\hat{e}, y^1, c^1)$ over his/her equilibrium bundle (e^1, y^1, c^1). Now, suppose that $c^1 - g(\hat{e}) > c^2 - g(e^2)$, namely that type 2 also strictly prefers the bundle $(\hat{e}, y^1, c^1)$ over his/her equilibrium bundle (which is (e^2, y^2, c^2)). Letting the subset S be given by $\{1, 2\}$, it follows that y^1 is the maximum income level associated with the effort level $\hat{e}$ in order for the firm not to make negative profits. Moreover the answer for question (b) is affirmative and condition (c) is vacuously satisfied (as there are no types s which are not in S). Thus, our suggested equilibrium would fail to satisfy the extended intuitive criterion.

It is straightforward to verify that given the income tax schedule, which confines the set of (gross) income levels to the pair (y^1, y^2), the deviation analyzed above is the only threat that challenges the separating equilibrium defined in problem $\mathcal{P}1$.[16] We thus refine the separating equilibrium, accommodating the extended intuitive criterion, by considering the separating tax

[13]Our exposition of the Grossman and Perry (1986) criterion follows Riley (2001).

[14]Notice that in case S is confined to be a singleton, we obtain the standard intuitive criterion.

[15]For example, the government can levy a 100% confiscatory tax rate at income levels different from y^1 and y^2.

[16]To see this, notice that bunching at the higher (gross) income level y^2 would dictate, by the zero profit condition, an effort level higher than e^2, which would make it an unattractive deviation for both types of workers.

equilibrium defined by the following constrained optimization problem, henceforth problem $\mathcal{P}2$:

Problem $\mathcal{P}2$

$$\max_{\{y^1,c^1,y^2,c^2\}} \quad c^1 - kg(e^1)$$

subject to:

$$y^i = e^i\theta^i, \quad i = 1,2 \tag{9}$$

$$c^1 - kg(e^1) \geq c^2 - kg(e^2), \tag{10}$$

$$c^2 - g(e^2) \geq c^1 - g\left(\frac{y^1}{\sum_i \gamma^i\theta^i}\right), \tag{11}$$

$$\sum_i \gamma^i(y^i - c^i) = 0. \tag{12}$$

Three remarks are in order. First, notice that the IC constraint given by (11) implies the weaker constraint given by (7) and hence renders the latter redundant. The fact that (11) is a tighter constraint than (7) reflects the presence of an information rent (obtained by type-2 agents) which is associated with the difference in innate abilities. If these were equal, there would be no difference between the two constraints. Second, notice that the separating allocation obtained under the extended intuitive criterion coincides with the Rothschild and Stiglitz (1976) separating equilibrium in the screening game, in which the first movers are the firms rather than the workers (who are the first movers in the signaling game). Finally, whereas only one of the IC constraints (6)-(7) (the one associated with the high-skilled worker) is binding at an optimum, it is straightforward to find parameter values such that the modified conditions in (10) and (11) both bind at the same time. This implies a potential violation of the standard zero marginal tax at the top property (Sadka 1976) which is due to a problem of adverse selection in the labor market.

To get an intuition why constraints (10) and (11) both could be binding under an optimal nonlinear income tax, recall that, under laissez-faire, low-skilled agents may find it attractive to replicate the effort of high-skilled agents in order to receive a higher remuneration (per unit of effort). Given that, in the presence of a government redistributing from high- to low-skilled workers, the downward IC constraint will always be binding (provided the social welfare function is egalitarian), the question is whether it is still attractive for type 1 agents to mimic type-2 agents in the presence of the redistribution carried out through the tax/transfer-system. In problem $\mathcal{P}1$, the answer is unambiguously negative, by virtue of the single-crossing property. However, in the re-formulated problem $\mathcal{P}2$, a mimicking type-2 agent would derive an information rent associated with the difference in productivities (reflected in the fact that the bundle associated with the type-2 mimicker differs from the equilibrium bundle intended for type-1 agents). If the productivity difference is large enough, it will render the bundle offered to type-2 agents sufficiently attractive for type-1 agents, implying that both IC conditions will be binding in the optimal solution.

2.2.2 A pooling equilibrium

By offering a single income level on the income tax schedule, denoted by $\hat{y}$, the government can implement a pooling equilibrium, as it can rule out deviations to income levels other than $\hat{y}$.[17] The reason pooling can be desirable in our setting is the possibility to redistribute via the wage channel, something that is infeasible in a standard optimal income tax model. In a pooling equilibrium, all workers choose the same level of effort $\hat{e}$. Hence, total production is equal to $\hat{y} = \hat{e}\overline{\theta}$, where $\overline{\theta} \equiv \sum_i \gamma^i \theta^i$ denotes the average innate ability. A government seeking to implement a pooling equilibrium will then solve the following problem, henceforth problem $\mathcal{P}3$:

Problem $\mathcal{P}3$

$$\max_{\hat{y}} \quad \hat{c} - kg(\hat{e}) \quad \text{subject to} \quad \hat{y} = \hat{e}\bar{\theta}, \quad \hat{c} = \hat{y}.$$

Note that pooling is not subject to cream-skimming and is hence stable as the income tax induces full compression of the income distribution (on- and off-equilibrium).

2.2.3 Comparison between separating and pooling equilibria

We now turn to shed some light on the forces which determine whether the separating or the pooling tax equilibrium constitutes the social optimum. As we will show, the separating equilibrium tends to dominate when: i) the difference in the cost of acquiring the signal is large (k is large); ii) θ^1 is either close to θ^2 or close to zero; iii) the distribution of types is very asymmetric ($|\gamma^1 - \gamma^2|$ is large).

We begin by considering the distortions that characterize the pooling equilibrium. From the first order condition of problem $\mathcal{P}3$, one obtains $kg'\left(\hat{y}/\bar{\theta}\right) = \bar{\theta}$, where $\hat{e} = \hat{y}/\bar{\theta}$ is the common effort level of both agents. This implies that, in a pooling tax equilibrium, the effort choice of type-1 agents is distorted upwards (since for them the efficient effort would satisfy $kg'(e) = \theta^1$) and the effort choice of type-2 agents is distorted downwards (since for them the efficient effort would satisfy $g'(e) = \theta^2$). We can also notice that the downward distortion faced by high-skilled agents is more severe the larger is k.

Consider next the optimal separating equilibrium. Here, instead, the effort choice of type-1 agents is distorted downwards, whereas the effort choice of type-2 agents is either left undistorted or distorted upwards. To see this, consider the definition of the optimal separating tax equilibrium (i.e., problem $\mathcal{P}2$) and denote by λ^1 the Lagrange multiplier attached to the constraint (10), and by μ the multiplier attached to the constraint (12). From the first-order conditions we obtain:

Proposition 1. *Under a separating equilibrium, the optimal distortions (wedges) in the scenario*

[17] The possibility to implement a pooling allocation in a two-type optimal income tax model without adverse selection in the labor market was discussed by (Stiglitz 1982). However, in that setting, pooling is Pareto-inferior to the laissez-faire allocation.

with only an income tax in place are given by the following expressions:

$$\tau_y^1 \equiv 1 - MRS_{yc}^1 = 1 - \frac{kg'\left(y^1/\theta^1\right)}{\theta^1} = \frac{\gamma^2 + \lambda^1}{\gamma^1} \frac{g'\left(y^1/\theta^1\right)}{\theta^1} \left[k - \frac{\theta^1}{\overline{\theta}} \frac{g'\left(y^1/\overline{\theta}\right)}{g'\left(y^1/\theta^1\right)} \right] > 0, \quad (13)$$

$$\tau_y^2 \equiv 1 - MRS_{yc}^2 = 1 - \frac{g'\left(y^2/\theta^2\right)}{\theta^2} = -\frac{\lambda^1}{\gamma^2} \frac{g'\left(y^2/\theta^2\right)}{\theta^2} (k-1) \leq 0. \quad (14)$$

Proof See Appendix A. □

The RHS of (13) provides a measure of the (downward) distortion imposed on y^1 and is unambiguously positive as $\frac{\theta^1}{\overline{\theta}} \frac{g'\left(e^1\theta^1/\overline{\theta}\right)}{g'(e^1)} < 1$ and $k > 1$. It reflects the fact that, given our max-min social welfare function, the constraint (11) will necessarily be binding, warranting a downward distortion on type-1 agents to deter mimicking by type-2 agents. The RHS of (14) provides a measure of the distortion imposed on y^2 and will either be zero (if constraint (10) is slack, i.e. $\lambda^1 = 0$) or it will be negative (if constraint (10) is binding, i.e. $\lambda^1 > 0$). If under laissez-faire the constraint (4) is slack, then the constraint (10) will necessarily be slack at an optimal separating tax-equilibrium, implying $\tau_y^2 = 0$. Intuitively, if low-skilled agents had no incentive to mimic high-skilled agents under laissez-faire, this will be all the more true under a redistributive policy which is designed to benefit low-skilled agents. Instead, if under laissez-faire the constraint (4) is binding, then the constraint (10) may be binding, in which case we have $\tau_y^2 < 0$.

On the one hand, the fact that redistribution is geared towards type-1 agents alleviates the incentives for them to mimic type-2 agents. On the other hand, at a separating equilibrium, type-2 agents enjoy an information rent associated with differences in productivities, and this information rent limits the generosity of the redistribution towards type-1 agents. If too little redistribution can be achieved, type-1 agents will still have an incentive to mimic type-2 agents. The bottom line of the discussion above is that, if the difference in the cost of signaling is sufficiently small (i.e., k is not too large) and the information rent (enjoyed by type-2 agents) associated with differences in productivities sufficiently large, a separating tax-equilibrium will also distort the behavior of type-2 agents.[18] When this is the case, both tax-equilibria (separating and pooling) will entail a double distortion, on the low- as well as on the high-skilled agents.

Let us now compare the two types of equilibria in terms of equity considerations.

Proposition 2. *The equity gains of switching from a separating to a pooling tax equilibrium are smallest when: i) θ^1 is close to θ^2, ii) θ^1 is close to zero, and, iii) $|\gamma^1 - \gamma^2|$ is large.*

Proof The equity gains of a pooling equilibrium are due to the elimination of the information rent, associated with the difference in productivities, that is enjoyed by high-skilled agents at a separating equilibrium. This information rent is proportional to the difference $\left(y_{sep}^1/\theta^1\right) - \left(y_{sep}^1/\overline{\theta}\right)$, i.e. to the difference between the effort exerted in equilibrium by type-1 agents and the

[18] When k is small enough, the constraint (4) is binding in laissez-faire.

effort that would be exerted, off-equilibrium, by type-2 agents behaving as mimickers, and can equivalently be expressed as

$$\frac{y_{sep}^1}{\theta^1} - \frac{y_{sep}^1}{\bar{\theta}} = \frac{\theta^2 - \theta^1}{\bar{\theta}\theta^1}\gamma^2 y_{sep}^1. \tag{15}$$

Regarding i), it is clear that when $\theta^1 \to \theta^2$, the RHS of (15) tends to zero. Regarding ii), notice as $\theta^1 \to 0$, the efficient level of e^1 (and y^1) approaches zero. Given that the optimal separating tax equilibrium distorts y_{sep}^1 downwards, it follows that, when $\theta^1 \to 0$, we have that $y_{sep}^1 \to 0$, implying that the RHS of (15) tends to zero. Regarding iii), consider first the case $\gamma^2 \to 0$. In this case, it is apparent that the RHS of (15) tends to zero (notice also that $\gamma^2 \to 0$ implies $\theta^1 \to \bar{\theta}$). Consider next $\gamma^2 \to 1$. In this case, the RHS of (15) tends to zero because $\gamma^2 \to 1$ implies $y_{sep}^1 \to 0$. The latter can be seen from the first order conditions of the government's problem in Appendix A. Combining (A 8) and (A 9) for $\gamma^2 \to 1$ (i.e. $\gamma^1 \to 0$) yields $\lambda^2 k g'\left(y^1/\theta^1\right) = \lambda^2\theta^1 g'\left(y^1/\bar{\theta}\right)/\bar{\theta}$, which can only be satisfied (taking into account that $\lambda^2 > 0$) for $y^1 = 0$. $\square$

It is easy to come up with numerical examples where pooling forms the social optimum when the parameters are such that the separating allocation induces a double distortion and the information rent of high-skilled mimickers is large. Moreover, when the number of types is greater than two, there is the possibility of socially optimal hybrid tax equilibria, featuring bunching, in which both predistribution and redistribution take place at the same time.[19]

2.3 The extended tax regime

We now extend our tax base and allow the government to tax each individual based on two observable characteristics: (i) earned income, (ii) the signal transmitted in the labor market. Thus, the tax function that we consider is a nonlinear and nonseparable function of both y and e, i.e. $T(y, e)$. We maintain all our earlier assumptions and in particular the asymmetry in information between firms and workers with respect to the latter's innate ability. Moreover, we focus on the separating equilibrium as the pooling equilibrium, in the extended tax regime, is identical to the one under the income tax regime.[20] The tax equilibrium in the extended tax regime is given by the solution to the following optimization problem, henceforth problem $\mathcal{P}4$:

[19]As pointed out earlier, there is an equivalence between the signaling model (subject to the extended intuitive criterion) and the competitive screening setup (see also Riley 2001, pp. 445-46). Therefore, we can refer to the numerical examples in Bastani et al. (2015) (see Figure 1 for the two-type case, and Figure 3 in Appendix E for the three-type case).

[20]Either an income tax or a tax on the signal can be used to implement a given pooling equilibrium.

Problem $\mathcal{P}4$

$$\max_{\{y^1,c^1,e^1,y^2,c^2,e^2\}} \quad c^1 - kg(e^1)$$

subject to:

$$y^i = e^i\theta^i, \quad i = 1, 2, \tag{16}$$

$$c^1 - kg(e^1) \geq c^2 - kg(e^2), \tag{17}$$

$$c^2 - g(e^2) \geq c^1 - g(e^1), \tag{18}$$

$$\sum_i \gamma^i(y^i - c^i) = 0. \tag{19}$$

Comparing the constraints (16)-(19) with the corresponding constraints under the income tax regime, i.e. constraints (9)-(12), one can see that the difference between the two regimes is confined to the IC constraint associated with the high-skill type. The possibility to tax e prevents agents from choosing effort levels other than those specified in the bundles (y^1, c^1, e^1) and (y^1, c^2, e^2). This implies that the stability threat posed by the extended intuitive criterion is no longer relevant: a type-2 worker behaving as a mimicker, i.e. earning y^1 in order to benefit from a more lenient tax treatment, is forced to choose the same effort level prescribed in equilibrium for a type-1 worker. In the extended tax regime, high-skilled agents no longer enjoy an information rent associated with the differences in productivities, and the amount of achievable redistribution is solely determined by the differences in the costs of acquiring the signal.

By alleviating one of the IC constraints that limit the amount of feasible redistribution, the extended tax regime allows to increase social welfare relative to the case with only an income tax in place. This result, which has important policy implications, is formally stated in Proposition 3 below. Proposition 3 also provides two additional results. First, it shows that the possibility to tax the signal renders a pooling equilibrium socially undesirable for any difference in productivities between the two types of workers and independently of the difference in the acquisition cost of the signal. This result, which is in contrast to what was the case under the optimal income tax regime, is related to the fact that, by replacing the RHS of (11) with the RHS of (18), we obtain a set of incentive constraints that cannot both be binding at the same time, namely, an identical set of constraints as (5)-(8). Second, the proposition shows that the optimal distortions are unrelated to the differences in productivities and, in particular, the distortion on type 2 agents is zero.

Proposition 3. *In the extended tax regime, the following holds:*

(i) the social optimum is always attained by a separating allocation, namely, predistribution is never optimal;

(ii) social welfare is strictly higher than under the optimal income tax regime;

(iii) the optimal distortions (wedges) in the extended tax regime take the form:

$$\tau_y^1 \equiv 1 - \frac{kg'\left(y^1/\theta^1\right)}{\theta^1} = \frac{\gamma^2}{\gamma^1}\frac{g'\left(y^1/\theta^1\right)(k-1)}{\theta^1} = \frac{\gamma^2}{k-\gamma^2}(k-1) > 0, \tag{20}$$

$$\tau_y^2 \equiv 1 - \frac{g'\left(y^2/\theta^2\right)}{\theta^2} = 0, \tag{21}$$

and are thus unrelated to the differences in productivities. Moreover, τ_y^1 is strictly smaller, and τ_y^2 is weakly smaller, than in the separating equilibrium with only an income tax.

Proof See Appendix B. □

Notice that, by conditioning the tax on both income and the signal, the information rents associated with the differences in productivities are eliminated. This is due to the fact that the extended tax regime blocks off-equilibrium deviations to education levels different than e^1 and e^2. In this way, type 2 mimickers are denied any information rent associated with their higher productivity when they choose to replicate the income level of low-skilled agents. Thus, an extended tax regime ameliorates the efficiency of the redistribution that can be carried through the income channel.[21] A reflection of this increased efficiency is that, at the optimal separating tax equilibrium under an extended tax regime, e^2 is always undistorted and e^1 is always less distorted than at the optimal separating tax equilibrium under the income tax regime. In particular, whereas both regimes imply a downward distortion on e^1 (i.e., $1-kg'\left(y^1/\theta^1\right)/\theta^1 = 1-kg'\left(e^1\right)/\theta^1 > 0$), under an extended tax regime e^1 is closer to its efficient value given by $g'^{-1}\left(\theta^1/k\right)$.[22]

Notice also that, as an alternative to supplementing the nonlinear income tax with a nonlinear tax on education, the government could set a binding education mandate (a lower bound) at the level of education associated with type-1 workers under the separating optimal allocation in the extended tax regime. By doing so, type-2 mimickers would be denied the information rent associated with the difference in productivities (as the mandate blocks the possibility for type-2 mimickers to credibly signal their superior productive ability to the firm, conditional on the income level of type-1 agents). Our analysis therefore suggests a novel normative justification for education mandates in the form of compulsory minimum schooling.[23]

The qualitative features of Proposition 3, which focuses on the two-type case, generalize to the case with $N > 2$ types. In particular, a pooling allocation is never optimal and social welfare is strictly higher than under the optimal income tax regime. Moreover, one can show that some of the bunching configurations that are optimal (depending on the specific parametric assumptions) under the income tax regime, become sub-optimal in the extended tax regime, implying that the

[21] In contrast, under a pooling equilibrium the high-skilled agents are always denied any information rents associated with difference in productivities, independently on whether income taxation is supplemented or not with a tax on the signal. Thus, when attention is confined to pooling equilibria, an extended tax regime does not improve upon the income tax regime.

[22] See part (iii) of Appendix B.

[23] Such policy measures are often warranted on efficiency grounds (internalization of positive spillovers, mitigating imperfections in capital markets etc.), see, e.g., Eckstein and Zilcha (1994) and Balestrino et al. (2016).

extended tax regime generally weakens the case for bunching.[24] However, whereas a separating equilibrium is always optimal in the two-type case, for $N > 2$ the optimality of a fully separating equilibrium hinges on the fulfillment of a necessary and sufficient condition. Denoting by k^i the cost of acquiring education for type-i agents and by γ^i their proportion in the population, this condition requires that, $\forall i \in \{1, ..., N-1\}$:

$$\frac{\gamma^{i+1}\theta^{i+1}}{\gamma^{i+1}k^{i+1} + (k^{i+1} - k^{i+2})\left(\sum_{j\geq i+2}\gamma^j\right)} > \frac{\gamma^i\theta^i}{\gamma^i k^i + (k^i - k^{i+1})\left(\sum_{j\geq i+1}\gamma^j\right)}. \tag{22}$$

A heuristic derivation of condition (22) is presented in Appendix D. When condition (22) is satisfied, a fully separating equilibrium is optimal and one obtains the following generalized version of (20), which applies for $i = 1, \dots, N$:[25]

$$\tau_y^i \equiv 1 - \frac{k^i g'\left(y^i/\theta^i\right)}{\theta^i} = \frac{\sum_{j\geq i+1}\gamma^j}{\gamma^i}\frac{g'(y^i/\theta^i)(k^i - k^{i+1})}{\theta^i} = \frac{(k^i - k^{i+1})\left(\sum_{j\geq i+1}\gamma^j\right)}{(k^i - k^{i+1})\left(\sum_{j\geq i+1}\gamma^j\right) + \gamma^i k^i}. \tag{23}$$

The first thing to notice about (23) is that, for $i = N$, it implies that $\tau_y^N = 0$ (given that $\sum_{j\geq N+1}\gamma^j = 0$) and that for $i = 1, ..., N-1$ it implies that $0 < \tau_y^i < 1$. The second thing to notice about (23) is that it confirms the property of Proposition 3 that the optimal distortions are unrelated to the differences in productivities. The intermediate step in (23) highlights that the optimal marginal tax rates are increasing with respect to both the inverse hazard rate of the skill distribution and the information rent associated with the differences in the costs of acquiring the signal across adjacent types. The final step in (23) highlights that the marginal tax rate faced by agents of type i is increasing in $(k^i - k^{i+1})\left(\sum_{j\geq i+1}\gamma^j\right)$. As we discuss in the Appendix D, this term represents the amount of resources that would be needed to preserve IC if type-i agents were induced to marginally raise their effort.

The general lesson from Proposition 3, and its extension to the $N > 2$ case, is that when signaling occurs in one dimension, and this dimension of signaling can be taxed, the case for predistribution, i.e. redistribution through the wage channel, is weakened. This is because the extended tax regime, by taxing the signal *conditional* on income, enhances the efficiency of redistribution through the income channel.

3 A model with signaling in two dimensions

We now extend the setting analyzed in section 2 by considering the availability of two signals. The first signal is denoted by e_s, and represents the quantity of effort. The second signal is

[24] For example, in the three-type case, one can show that bunching involving types 2 and 3 will never be an optimal solution under the extended tax regime, whereas it will be optimal in the income tax regime under certain parametric assumptions.

[25] The full derivations are skipped for the sake of brevity and are available upon request.

denoted by e_q, and represents the intensity of effort. For instance, in the context of education, the variables e_s and e_q would capture, respectively, the quantity (e.g., the time spent in acquiring vocational training and/or academic degrees) and quality (e.g., GPA, reputation of certifying institute, etc.) dimensions of educational attainment.[26] We assume that e_s is observed by both the government and the firms, whereas e_q is only observed by the firms (or prohibitively costly to observe by the government). The output of a type-i worker is given by the production function:

$$z^i = h(e_s^i, e_q^i)\theta^i,$$

where $h(\cdot)$ is jointly concave and strictly increasing in both arguments. The utility function is:

$$u^i(c, e_s.e_q) = c - R^i(e_s, e_q),$$

where c denotes consumption and

$$R^i(e_s, e_q) = p_s^i e_s + p_q^i e_q$$

is the cost function for type-i agents, with p_s^i and p_q^i denoting, respectively, the unitary marginal cost of e_s and the unitary marginal cost of e_q for an agent of type i.[27] We will hereafter make the following assumptions:

$$p_s^1 = p_s^2 \equiv p_s \quad \text{and} \quad p_q^1 > p_q^2,$$

which jointly imply that type-2 agents have a (weak) absolute advantage in signaling via each channel, and a comparative advantage in the quality signal e_q. Before turning to the government problem, we define the laissez-faire.

3.1 Laissez-faire market equilibrium

The two-stage signaling game in the presence of two-dimensional signals works as follows. In the first stage, workers choose their levels of effort (both quality and quantity components), (e_s^i, e_q^i); $i = 1, 2$. In the second stage each firm offers a labor contract which specifies the income level (which is also the consumption level with no taxes/transfers in place) as a function of the observed signals, namely, $y(e_s, e_q)$. Based on the observed signals, firms form their beliefs with respect to the workers' types. In equilibrium, choices are consistent in the sense that firms maximize their expected profits by choosing the labor contracts, given their beliefs; and, workers maximize their utility (by choosing their signal effort-levels) given the labor contracts offered by the firms. Assuming a perfectly competitive labor market with free entry of firms implies that

[26]In the context of labor supply, e_s and e_q would for instance capture, respectively, the time and intensity components of labor effort.

[27]Notice, that in the model considered in this section, the nonlinearity in the utility maximization problem derives from the production function, whereas in (1) it derived from the effort cost function. However, the two formulations are iso-morphic.

rents are fully dissipated, hence, $y^i = h(e_s^i, e_q^i)\theta^i$.

The laissez-faire equilibrium (assuming it exists, see section 2.1) is given by the pairs (e_s^{i*}, e_q^{i*}), $i = 1, 2$, satisfying the following conditions:

$$(e_s^{1*}, e_q^{1*}) = \underset{e_s^1, e_q^1}{\operatorname{argmax}} \Big\{ h(e_s^1, e_q^1)\theta^1 - R^1(e_s^1, e_q^1) \Big\}, \tag{24}$$

and

$$(e_s^{2*}, e_q^{2*}) = \underset{e_s^2, e_q^2}{\operatorname{argmax}} \Big\{ h(e_s^2, e_q^2)\theta^2 - R^2(e_s^2, e_q^2) \Big\} \tag{25}$$

subject to:

$$h(e_s^{1*}, e_q^{1*})\theta^1 - R^1(e_s^{1*}, e_q^{1*}) \geq h(e_s^2, e_q^2)\theta^2 - R^1(e_s^2, e_q^2), \tag{26}$$

which has a similar structure as the definition of the laissez-faire equilibrium in the case of signaling in one dimension. The above IC constraint reflects the fact that type-1 workers might have an incentive to mimic their higher-skilled counterparts in order to be remunerated according to the (higher) productivity of type-2 agents. If the constraint is binding, an adverse selection problem arises, implying that type-2 workers are induced to over-invest in the quality-signal e_q (due to their comparative advantage in this dimension of signaling).

3.2 The income tax regime

We first consider the benchmark setup where an individual's tax liability is just a function of his/her earned income. We assume, like we did for the case of signaling in one dimension, that the government is invoking a max-min social welfare function. The income tax is, as before, defined by a set of pre-tax/post-tax income bundles denoted by (y^i, c^i) where the total tax (or transfer if negative) is defined by $t^i \equiv y^i - c^i$. We define the wage rate earned by a given individual as the ratio between his/her pre-tax income y and the value of the h-function evaluated at the effort vector chosen by the individual, thereby generalizing the definition of wage rate provided in Section 2.2.

3.2.1 A separating tax equilibrium

Under a separating equilibrium, the tax schedule is designed in such a way to induce type-1 agents to select a bundle (y^1, c^1) with associated tax payment $t^1 = y^1 - c^1 < 0$, and to induce type-2 agents to select a bundle (y^2, c^2) with associated tax payment $t^2 = y^2 - c^2 > 0$. Type-i agents choosing the bundle intended for them on the tax schedule would choose an efficient mix of e_s and e_q, denoted by $(e_s^i(y^i), e_q^i(y^i))$, $i = 1, 2$, with an associated cost given by:

$$R^i(y^i) = \min_{e_s, e_q} R^i(e_s, e_q) \quad \text{subject to} \quad h(e_s, e_q)\theta^i = y^i. \tag{27}$$

Notice that efficiency in the choice of the effort mix means that $e_s^i(y^i)$ and $e_q^i(y^i)$ satisfy the condition

$$\frac{\partial h\left(e_s^i(y^i), e_q^i(y^i)\right)/\partial e_s^i}{\partial h\left(e_s^i(y^i), e_q^i(y^i)\right)/\partial e_q^i} = \frac{p_s}{p_q^i},$$

which equates the marginal rate of technical substitution (MRTS) to the marginal cost ratio. Moreover, notice that under a separating equilibrium, agents are paid by the firm according to their true productivity (a type i agent is paid a wage rate of θ^i).

To implement a given separating equilibrium, the government has to guard against various deviating strategies available to agents, i.e. the government has to make sure that no agent has an incentive to deviate from the behavior expected from him/her. There are in principle three deviating strategies that may be adopted by an agent of type i choosing to earn the income y^j intended for the other type. The agent can choose an effort vector that enables him/her to get remunerated according to: (i) the productivity of the other type, (ii) the average productivity, or, (iii) his/her true productivity. We consider these three deviating strategies in more detail below. Given that a deviating agent is someone who earns an amount of income which is intended for some other type of agents, we will follow the common practice of using the word "mimicker" to refer to a deviating agent in all three cases.

A first deviating strategy is for type-i agents to earn the income level y^j by choosing the effort mix $(e_s^j(y^j), e_q^j(y^j))$ chosen in equilibrium by type-j agents. Behaving in this way, a type-i mimicker would be paid a wage rate θ^j (i.e., according to the productivity of the type being mimicked) and would incur the following cost:

$$\breve{R}^i(y^j) = p_s^i \breve{e}_s^i(y^j) + p_q^i \breve{e}_q^i(y^j), \tag{28}$$

where $(\breve{e}_s^i(y^j), \breve{e}_q^i(y^j)) = (e_s^j(y^j), e_q^j(y^j))$ denotes the effort mix of a type i mimicker, which is identical to the effort mix chosen in equilibrium by agents of type j. Notice that $(\breve{e}_s^i(y^j), \breve{e}_q^i(y^j))$ is, from the perspective of type-i agents, a distorted effort mix, in the sense that it does not satisfy the condition $\frac{\partial h(e_s,e_q)/\partial e_s}{\partial h(e_s,e_q)/\partial e_q} = \frac{p_s}{p_q^i}$.

Besides the deviating strategy described above, which involves a type-i mimicker choosing the effort vector selected in equilibrium by agents of type $j \neq i$, there are also deviating strategies that involve the choice, by a mimicker, of an off-equilibrium effort vector.

The first of such strategies is the possibility for a type-i agent to earn the income level y^j by choosing an effort vector which is at the same time: i) different from the one chosen in equilibrium by type-j agents, ii) attractive also for type-j agents, and iii) sufficient to allow firms to make non-negative profits when remunerating agents according to the average productivity $\bar{\theta}$. For a type-i mimicker, the most attractive of such strategies is the one with associated cost given by:

$$\hat{R}^i(y^j) = \min_{(e_s,e_q)\neq(e_s^j(y^j),e_q^j(y^j))} R^i(e_s,e_q) \tag{29}$$

subject to:

$$R^j(e_s,e_q) \leq R^j(y^j), \tag{30}$$

$$y^j \leq h(e_s,e_q)\bar{\theta}. \tag{31}$$

The constraint (30) captures the fact that the deviating strategy is feasible insofar as it induces also type-j agents to change their effort vector. The constraint (31) ensures that the effort vector is enough to deliver a non-negative profit for the hiring firm in a pooling equilibrium where both agents are paid according to the average productivity $\bar{\theta}$. Lemma 1 shows that this off-equilibrium deviation is only feasible for type 2 agents.

Lemma 1. *Type-1 agents cannot succeed in earning y^2 while being remunerated according to the average productivity $\bar{\theta}$.*

Proof Under the suggested deviating strategy both types of workers would earn y^2 while being paid according to the average productivity $\bar{\theta}$ and exerting the same effort vector (e_s, e_q) satisfying $h(e_s,e_q)\bar{\theta} \geq y^2$. However, recalling that the equilibrium effort vector chosen by type-2 agents at the income level y^2 is efficient, type 2 agents cannot be induced to prefer such an allocation. The reason is that, since they would be remunerated according to $\bar{\theta}$ rather than according to their true productivity θ^2, they would necessarily be forced to adopt a more costly effort vector.[28] □

The next Lemma shows that the off-equilibrium strategy with cost $\hat{R}^i(y^j)$ is always superior (in the sense of being less costly) for type-2 agents relative to the mimicking strategy of replicating the effort vector chosen in equilibrium by agents of type 1.

Lemma 2. *For a type-2 agent, it is always more attractive to earn y^1, while being remunerated according to the average productivity $\bar{\theta}$, compared to earning y^1, while being remunerated according to the low productivity $\theta^1 < \bar{\theta}$. In other words, $\hat{R}^2(y^1) < \breve{R}^2(y^1)$.*

Proof Let $(e_s^1(y^1), e_q^1(y^1))$ denote the effort vector chosen by type-1 agents at the bundle intended for them by the government, and let $\bar{e}_s = e_s^1(y^1) - \epsilon$ and $\bar{e}_q = e_q^1(y^1) - \epsilon$, for small $\epsilon > 0$, represent a candidate effort vector for a type-2 mimicker. As $\bar{\theta} > \theta^1$ and $y^1 = h(e_s^1(y^1), e_q^1(y^1)) \cdot \theta^1$, it follows by continuity that $h(\bar{e}_s, \bar{e}_q) \cdot \bar{\theta} > y^1$. Hence, the suggested effort vector does not violate the constraint requiring firms to make non-negative profits. By construction, $R^2(\bar{e}_s, \bar{e}_q) < \breve{R}^2(y^1)$ and $R^1(\bar{e}_q, \bar{e}_s) < R^1(y^1)$, so the candidate effort vector is preferred by both types of workers and induces pooling. Moreover, by virtue of the fact that $\hat{R}^2(y^1)$ represents the minimal cost for type 2 under a pooling equilibrium, we have that $\hat{R}^2(y^1) \leq R^2(\bar{e}_s, \bar{e}_q)$. Thus, it follows that

[28] Notice that, if the equilibrium effort vector chosen by type-2 agents at the income level y^2 were not efficient, one could no longer rule out the possibility that type-1 agents succeed in earning y^2 while being remunerated according to the average productivity $\bar{\theta}$.

$\hat{R}^2(y^1) < \breve{R}^2(y^1)$. This completes the proof. $\square$

The other deviating strategy that involves the choice of an off-equilibrium effort vector is the one where type-i agents replicate the earned income y^j of type-j agents, but invest in the signals in such a way so as to separate themselves from type j agents, thereby succeeding in being remunerated by firms according to their true productivity θ^i. For a type-i mimicker, the most attractive of such strategies is the one with associated cost given by:

$$\tilde{R}^i(y^j) = \min_{(e_s,e_q)\neq(e_s^j(y^j),e_q^j(y^j))} R^i(e_s, e_q) \tag{32}$$

subject to:

$$R^j(e_s, e_q) \geq R^j(y^j), \tag{33}$$

$$y^j \leq h(e_s, e_q)\theta^i. \tag{34}$$

In the problem above, constraint (33) ensures that the effort vector chosen by type-i mimickers is not attractive for type-j agents, thereby allowing type-i mimickers to separate themselves from their type-j counterparts. The constraint (34) ensures instead that the effort vector chosen by type-i mimickers is sufficient to produce y^j. Notice that, since $\theta^2 > \theta^1$, and given our assumptions that $p_s^2 = p_s^2 \equiv p_s$ and $p_q^1 > p_q^2$, it necessarily follows that $\tilde{R}^2(y^1) < R^1(y^1)$ and $\tilde{R}^1(y^1) > R^2(y^2)$. Notice also that there are two possible scenarios in which type-i agents succeed in separating themselves from type-j agents at the income level y^j: one in which constraint (33) is binding, and another in which it is slack. In the former case, the agent behaving as a mimicker will employ a distorted effort mix (i.e., an effort mix violating the condition $\frac{\partial h(e_s,e_q)/\partial e_s}{\partial h(e_s,e_q)/\partial e_q} = \frac{p_s}{p_q^i}$); in the latter case the effort mix chosen by the mimicker will be undistorted.

Using the definitions of R^i, $\breve{R}^i$, $\hat{R}^i$ and $\tilde{R}^i$, $i = 1, 2$ above, and Lemmas 1 and 2, we can state the optimal tax problem in the presence of two dimensions of signaling as Problem $\mathcal{P}5$:

Problem $\mathcal{P}5$ In a model with two dimensions of signaling with only an income tax in place, if the social optimum is a separating equilibrium, it is given by the solution to:

$$\max_{\{y^i,c^i\}_{i=1,2}} c^1 - R^1(y^1)$$

subject to the government budget constraint:

$$\sum_i \gamma^i(y^i - c^i) = 0,$$

and the IC constraints:

$$c^2 - R^2(y^2) \geq c^1 - \min\left\{\widetilde{R}^2(y^1), \hat{R}^2(y^1)\right\}, \tag{35}$$

$$c^1 - R^1(y^1) \geq c^2 - \min\left\{\breve{R}^1(y^2), \widetilde{R}^1(y^2)\right\}. \tag{36}$$

In Problem $\mathcal{P}5$, the expressions for $R^i(y^i), i = 1, 2$, are given by (27), the expression for $\breve{R}^1(y^2)$ is given by (28), the expression for $\hat{R}^2(y^1)$ is given by (29), and the expressions for $\widetilde{R}^i(y^j)$ are given by (32). The IC constraint for type-2 agents in (35) is naturally binding in the optimum, by virtue of the strictly egalitarian preferences exhibited by the government, whereas the IC-constraint for type 1-agents in (36) may be binding or not.

A type-i mimicker naturally opts for the least costly choice of the possibilities in curly brackets [as reflected by the $\min[\cdot]$ argument on the RHS of (35) and (36)]. Notice that under both the relevant deviating strategies, a type i mimicker obtains a consumption equal to c^j. Hence, determining which one of the two deviating strategies dominates requires only comparing the sustained effort cost under each of the two strategies. However, if $\min\left\{\breve{R}^1(y^2), \widetilde{R}^1(y^2)\right\} = \widetilde{R}^1(y^2)$, the constraint (36) will be necessarily slack. To see this, suppose that the social optimum is a separating equilibrium and that the constraint (36) is binding with $\min\left\{\breve{R}^1(y^2), \widetilde{R}^1(y^2)\right\} = \widetilde{R}^1(y^2)$. Given that $y^1 - c^1 < 0$ and $y^2 - c^2 > 0$, the government could then do better by removing (y^1, c^1) from the menu of bundles available on the income tax schedule and letting type-1 agents sustain the cost $\widetilde{R}^1(y^2)$ and pool with type-2 agents at (y^2, c^2). Type-1 agents would not suffer given that, by assumption, we had that $c^1 - R^1(y^1)) = c^2 - \widetilde{R}^1(y^2)$. At the same time, given that $y^1 - c^1 < 0$ and $y^2 - c^2 > 0$ at the supposedly optimal separating equilibrium, the government would experience a boost in revenue.

A final remark is in order. If the difference $p_q^1 - p_q^2$ is sufficiently large, we have that $\widetilde{R}^2(y^1) < \hat{R}^2(y^1)$. This can be seen by considering the limiting case where $p_q^1 \to \infty$ and $p_q^2 \to 0$, in which type-2 agents can separate themselves from type-1 agents (conditional on earning the same level of gross income y^1) by investing heavily in the quality component and incurring arbitrarily small costs. In this scenario, type-1 workers would entail arbitrarily large costs by following the same strategy, and hence would find it unattractive. Moreover, as compared to adopting a deviating strategy that is attractive to both types of agents, type 2 agents incur a lower effort cost when they replicate the earned income of type 1 agents, but invest in signals so as to distinguish themselves from their type 1 counterparts. This observation highlights a crucial difference between the current two-signal model and the previous one-signal model. The fact that there are two signals that can be used by type-2 workers, combined with the fact that type-2 workers have a comparative advantage (in the quality dimension) makes it possible (and under certain parametric assumptions optimal) for type-2 workers to separate themselves from their type-1 counterparts (conditional on choosing a given level of income). This could not be done with a single signal in place, as any reduction in the level of investment in the signal would be desirable for type-1 as well.[29]

3.2.2 A pooling tax equilibrium

As an alternative to a separating equilibrium the government might try to implement a pooling equilibrium. Given that there are no exogenous public revenue needs, under a pooling equilibrium, the income tax system offers the same pre-tax income $\hat{y}$ to both types of agents, which also coincides with the net income, denoted by $\hat{c}$. In the model with one dimension of signaling discussed in Section 2, pooling on income necessarily implied pooling on the signal. With two-dimensional signaling, instead, it is possible to have pooling on income without pooling in terms of the effort vectors chosen by the two agents. The following Proposition shows that such an equilibrium will never constitute the social optimum.

Proposition 4. *With two dimensions of signaling, and only an income tax in place, pooling on income without pooling on the effort signals observed by firms is socially suboptimal.*

Proof We begin by noting that pooling on income without pooling on the effort signals observed by firms entails no redistribution as both workers would have the same pre-tax income and would be remunerated according to the wage rates commensurate with their true ability. This implies that redistribution is carried out neither through the income channel nor through the wage channel. If the laissez-faire equilibrium is efficient, which happens when the IC constraint of type-1 agents in the laissez-faire is non-binding, pooling on income without pooling on signals would be Pareto-dominated by the laissez-faire allocation. The reason is that such a pooling allocation distorts the income/consumption bundle of at least one of the types without gaining anything on the equity side.[30] If, on the other hand, the IC constraint of type-1 agents is binding at the laissez-faire equilibrium, such that the effort choices of type-2 agents are distorted in the laissez-faire, it is possible that pooling on income could mitigate this distortion. However, since pooling on income without pooling on signals cannot achieve any redistribution, this efficiency gain would only benefit type-2 agents, and would hence not contribute to social welfare given our focus on the max-min social welfare function.[31] □

Given the result stated in Proposition 4, one can restrict attention to pooling equilibria where all agents choose the same effort vector. In this case, workers become indistinguishable from the

[29] Here we assume that the quantity marginal cost is the same for both types whereas the quality marginal cost faced by type-2 workers is lower. One could alternatively assume that both marginal costs are lower for type-2 and satisfy: $p_n^1 = kp_n^2$, where $k > 1$ and $n = s, q$. In such a scenario, type-2 would have an absolute advantage in acquiring the signal (thereby ensuring the single-crossing property, which is essential for separation). However, no comparative advantage emerges, as both types are faced with the same quality/quantity marginal cost ratio. In such a case separation (conditional on choosing a given level of income) is infeasible.

[30] The argument bears similarity to the standard argument why bunching is never optimal with two types in a standard Mirrleesian setup without asymmetric information between firms and workers, see e.g., Stiglitz (1982).

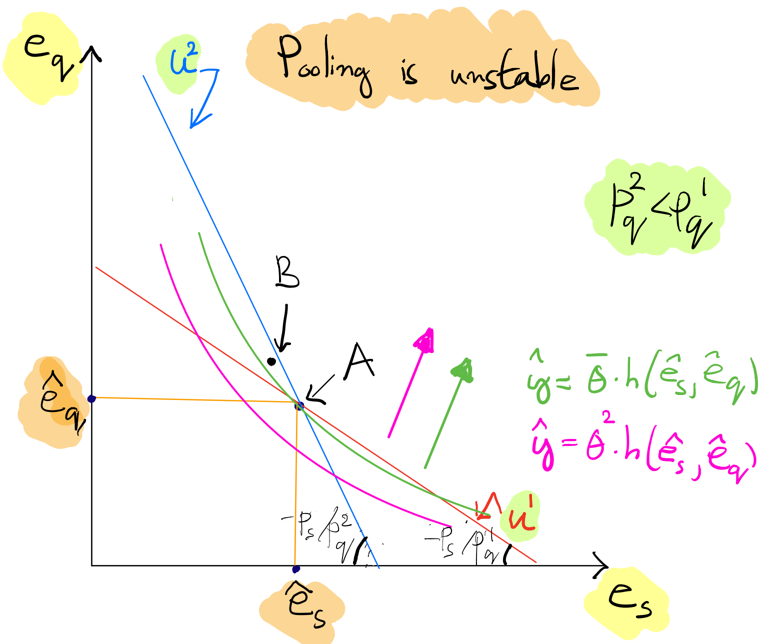
[31] With a social welfare function that attaches a positive weight to type-2 agents, the potential efficiency gain accruing to them would have to be weighed against the distortions imposed by the pooling allocation on the choices of type-1 agents. In this case, pooling on income without pooling on the observable effort signal can be ruled out by continuity provided the weight on type-2 agents is sufficiently small.

perspective of the firm and redistribution is accomplished through the wage channel (as both workers receive a wage rate equal to the average productivity $\bar{\theta}$). However, as Proposition 5 below shows, such a pooling equilibrium does not exist.

Proposition 5. *With two dimensions of signaling, and only an income tax in place, a pooling tax equilibrium where both workers choose the same effort vector does not exist.*

Proof Consider a candidate pooling allocation $(\hat{y}, \hat{c})$. By virtue of Proposition 4, we can restrict attention to situations where both workers choose the same effort mix, given by the tuple $(\hat{e}_s, \hat{e}_q)$. By construction, we have that $\hat{c} = \hat{y} = \bar{\theta} h(\hat{e}_s, \hat{e}_q)$, with $\bar{\theta} \equiv \sum_i \gamma^i \theta^i$. Let $\hat{u}^i = u^i(\hat{c}, \hat{e}_s, \hat{e}_q)$. Then, $\hat{c} - (p_s e_s^i + p_q^i e_q^i) = \hat{u}^i$, for $i = 1, 2$, will describe the indifference curves, in the (e_s, e_q) plane, going through the point $(\hat{e}_s, \hat{e}_q)$ indicated by A in figure 1. By virtue of our parametric assumptions, the indifference curve associated with type-2 workers is steeper than that associated with their type-1 counterparts. The intersection of the two downward sloping indifference curves creates a fork-shaped area to the north-west of point A in the figure. Now consider a slight shift from point A to B which lies within the aforementioned fork-shaped area. By construction, point B is preferred by type-2 workers to the pooling allocation (it lies below their indifference curve), whereas type-1 workers strictly prefer the pooling allocation A over the perturbed allocation, B (it lies above his indifference curve). Thus, by deviating to point B, type-2 workers credibly reveal their productivity to the firm and get remunerated accordingly. As $\theta^2 > \bar{\theta}$, it follows, by continuity, that the total output produced by a deviating type-2 worker would strictly exceed $\hat{y}$. Thus, the firm would find it profitable to hire the deviating type-2 worker. We have therefore established that the candidate pooling equilibrium is unstable.

Figure 1: Illustration of Proposition 5 and the non-existence of a pooling equilibrium



□

An immediate corollary of Proposition 4 and 5, that we summarize below, is that, under a pure income tax regime, the optimal solution is given by a separating equilibrium in which types 1 and 2 earn different levels of income. This represents a crucial difference between the model with signaling in two dimensions and the one-signal model considered in section 2.

Corollary 1. *With two dimensions of signaling, and only an income tax in place, the social optimum is always given by a separating tax equilibrium. Thus, predistribution is never optimal.*

3.3 The extended tax regime

We now extend our tax base and allow the government to tax each individual based on two observable characteristics: (i) earned income y, and, (ii) the quantity of effort e_s (e.g., years of education, hours physically spent at work). Recall that, by assumption, the intensity (quality) dimension of effort e_q is only observable by the firms, and hence, cannot be subject to taxation. An immediate and interesting implication of allowing to tax e_s is the feasibility of a pooling equilibrium.

The pooling equilibrium in the extended tax regime In the extended tax regime, if the social optimum is given by a pooling equilibrium (y, e_s), the level of social welfare is given by:

$$u^1 = y - R^1(e_s, \widehat{e}_q(y, e_s)), \tag{37}$$

where $\widehat{e}_q(y, e_s)$ is defined as the e_q which solves $y = h(e_s, e_q)\bar{\theta}$. Notice that the equity gains of pooling crucially depend on the difference in productivities between the two types of workers whereas the efficiency properties of a pooling equilibrium crucially depend on the differences in the costs of acquiring the signal e_q. Proposition 6 establishes the feasibility of pooling in the extended tax regime.

Proposition 6. *With two dimensions of signaling, the extended tax regime enables the government to implement a pooling tax equilibrium where both workers choose the same effort vector. Thus, predistribution becomes feasible.*

A formal proof is skipped for brevity purposes. However, Proposition 6 can easily be established noticing that an extended tax regime effectively confines signaling to one dimension, implying that cream-skimming is rendered infeasible, as in the setup with a single dimension of signaling.[32] We may also notice that supplementing the income tax with a mandate on e_s would represent an alternative way to allow implementing the pooling equilibrium. As high types would prefer to allocate more effort to the quality dimension, setting a mandate on the *quantity*

[32]One difference is that in the current setting, type-2 workers have a *comparative* advantage in e_q, which can be exploited by type-2 workers to separate themselves from their type-1, lower-skilled, counterparts. In the setting considered in section 2, high-skilled workers had an *absolute* advantage in acquiring the (single) signal e.

dimension implies that high-skill agents would be prevented from engaging in cream-skimming (identifying themselves as high types) while the effort mix of low-skilled agents would be left undistorted.

The separating equilibrium in the extended tax regime Let us now turn to the separating equilibrium. The first thing to notice is that the deviations associated with $\tilde{R}$ in constraints (35) and (36) become infeasible. For type-2 agents, the best deviating strategy will be to pool with type 1 agents, obtaining the wage rate $\bar{\theta}$. However, due to the fact that the tax is conditioned on both y and e_s, a type-2 mimicker is forced to replicate the quantity-signal e_s of type-1 agents (e_s^1), making mimicking less attractive. For type-1 agents, the only feasible deviating strategy is to replicate the effort choices of type-2 agents. Formally, if the social optimum in the extended tax regime is given by a separating equilibrium, it is given by the solution to Problem $\mathcal{P}6$:

Problem $\mathcal{P}6$

$$\max_{\{y^i,c^i,e_s^i\}_{i=1,2}} c^1 - R^1(e_s^1, e_q^{1*}) \tag{38}$$

subject to the government budget constraint:

$$\sum_i (y^i - c^i)\gamma^i = 0, \tag{39}$$

and the incentive constraints

$$c^2 - R^2(e_s^2, e_q^{2*}) \geq c^1 - R^2(e_s^1, \hat{e}_q^2) \tag{40}$$

$$c^1 - R^1(e_s^1, e_q^{1*}) \geq c^2 - R^1(e_s^2, e_q^{2*}) \tag{41}$$

where

$$\hat{e}_q^2 \quad \text{is the solution to} \quad y^1 = h(e_s^1, e_q^2)\bar{\theta} \tag{42}$$

$$e_q^{i*} \quad \text{is the solution to} \quad y^i = h(e_s^i, e_q^i)\theta^i, i = 1, 2. \tag{43}$$

The incentive constraint (40), together with the definition of $\hat{e}_q^2$ provided by (42), reflect that for type-2 mimickers the optimal deviating strategy is the off-equilibrium deviation where they pool with type-1 agents at the income level y^1. This is a feasible strategy since the effort vector $(e_s^1, \hat{e}_q^2)$ would certainly be attractive to type-1 agents due to the fact that, since $\bar{\theta} > \theta^1$, $\hat{e}_q^2 < e_q^{1*}$. The incentive constraint (41) reflects that the only feasible deviating strategy for type-1 mimickers is for them to earn y^2 by replicating the equilibrium effort choices of type-2 agents.

The social optimum The problem solved by a max-min government in the extended tax regime is to maximize the well being of type-1 workers, implementing either the optimal separating or the optimal pooling tax equilibrium, depending on which equilibrium configuration yields the highest utility to type-1 agents. A fundamental difference from the extended tax regime in the case of one signal (analyzed in section 2) is that, in the case of two signals, supplementing the income tax with a tax on the signal that is observed by the government is not enough to fully eliminate, under a separating equilibrium, the information rent associated with differences in productivities. As the improvement of the separating equilibrium offered by the extended regime is smaller when signaling occurs in two dimensions, relative to the case when signaling is confined to one dimension, the separating equilibrium does not necessarily dominate the pooling equilibrium from a social welfare perspective, in contrast to Proposition 3.[33] This insight is summarized in the following result:

Proposition 7. *With two dimensions of signaling and an extended tax regime in place, the pooling tax equilibrium can constitute the social optimum (in contrast to the case with only one dimension of signaling). Thus, predistribution is socially desirable.*

A formal proof is skipped for brevity purposes. Instead, we illustrate Proposition 7 through a numerical example in Section 4.4 where we show that either equilibrium configuration may arise as the social optimum depending on parametric assumptions. The comparison of the two types of equilibrium configurations is similar to that made in section 2.2.3. In particular, a separating equilibrium tends to dominate when: i) the difference $p_q^1 - p_q^2$ is large; ii) the gap in innate abilities ($\theta^2 - \theta^1$) is either small or θ^1 is close to zero; iii) the distribution of types is very asymmetric ($|\gamma^1 - \gamma^2|$ is large).[34]

The next Proposition sheds light on the efficiency properties of the separating versus pooling equilibrium by characterizing optimal distortions.

Proposition 8. *The optimal directions of distortions in the extended tax regime with two dimensions of signaling can be summarized as follows:*

(i) If the social optimum is a pooling equilibrium:

- e_s^1 *and* e_q^1 *are undistorted, whereas* e_s^2 *is distorted upwards and* e_q^2 *is distorted downwards.*
- y^1 *is distorted upwards and* y^2 *is distorted downwards.*

(ii) If the social optimum is a separating equilibrium:

- e_s^1 *is distorted upwards and* e_q^1 *is distorted downwards, whereas* e_s^2 *and* e_q^2 *are either undistorted, or* e_s^2 *is distorted downwards and* e_q^2 *upwards.*

[33] The pooling equilibrium fully eliminates the information rents associated with the differences in productivities by forcing wage equalization, but typically has less desirable efficiency properties.

[34] Notice that, in contrast to the case with signaling in one dimension, a mandate would not be sufficient to implement the extended tax regime if the social optimum is given by a separating equilibrium and the upward constraint (41) is binding.

- *y^1 is distorted downwards, whereas y^2 is either undistorted or distorted upwards.*

Proof See Appendix C. □

Notice that the pattern of distortions is coherent with the goal of relaxing binding IC constraints. Given that (40) is necessarily binding under a max-min objective, distorting the effort mix of type-1 agents towards the component e_s (in which type 1 agents have a comparative advantage) is instrumental in discouraging mimicking by type-2 agents. Conversely, distorting the effort mix of type-2 agents towards the component e_q (in which type 2 agents have a comparative advantage) is instrumental in discouraging mimicking by type-1 agents. However, given that constraint (41) may either be binding or not, distorting the effort mix of type-2 agents is only warranted when mimicking by type-1 agents is a relevant concern.

To keep our analysis tractable, we have restricted attention to a model with two types. The $N > 2$ case is more complex due to the nature of incentive compatibility constraints, both under the income tax regime and under the extended regime, reflecting the myriad of deviating profiles available. Some qualitative features are however worth stressing out. First, the possibility for cream-skimming implies that pooling or bunching is not feasible under an income tax regime, thereby ruling out redistribution through the wage channel. Second, under the extended tax regime, bunching and pooling become feasible and may dominate the fully separating regime.[35]

4 An illustrative example

In this section, we make a functional form assumption that allows us to shed more light on the IC constraints that are relevant in the bi-dimensional signaling case when the government aims at implementing a separating equilibrium. In particular, we assume:

$$h(e_s, e_q) = (e_s e_q)^\beta A, \tag{44}$$

where A is a positive constant and $0 < \beta < 1/2$. Assumption (44) will also be used to provide a numerical example of the results delivered by our model.

[35] We briefly comment on the case of more than two signals. In line with the analysis above, if the tax liability cannot be conditioned on (both income and) the entire set of signals, taxing the signals can only mitigate (but not fully eliminate) the information rent stemming from differences in productivities. Then, one might be tempted to argue that a pooling equilibrium might be socially desirable based on equity considerations and that the case for implementing a pooling equilibrium becomes stronger the smaller the set of signals on which the tax liability is conditioned. The problem with this reasoning is that it overlooks the fact that a pooling equilibrium must necessarily be sustainable in order to be implementable. In general, denoting by n the cardinality of the set of signals, pooling will be sustainable either when the (minimum) number of taxed signals is $n-1$, or when it is $n-j$ (with $1 < j < n$) and the high-skilled individuals have no comparative advantage within the set of signals unobserved (and therefore untaxed) by the government.

4.1 (In)efficiency of the laissez-faire

Consider what would be an efficient laissez-faire outcome if agents were remunerated based on their true productivities. A type i agent would then solve:

$$\max_{e_q,e_s} \quad (e_q e_s)^\beta A\theta^i - p_s e_s - p_q^i e_q.$$

Denoting by e_q^i and e_s^i the optimal value for the choice variables, from the first order conditions of the problem above one would get that

$$e_q^i = (p_s)^{-\frac{\beta}{1-2\beta}} \left(p_q^i\right)^{\frac{(\beta-1)}{1-2\beta}} \left(\beta A\theta^i\right)^{\frac{1}{1-2\beta}}, \tag{45}$$

$$e_s^i = (p_s)^{-\frac{1-\beta}{1-2\beta}} \left(p_q^i\right)^{-\frac{\beta}{1-2\beta}} \left(\beta A\theta^i\right)^{\frac{1}{1-2\beta}}, \tag{46}$$

implying that

$$U^i = \left(p_s p_q^i\right)^{-\frac{\beta}{1-2\beta}} \left(A\theta^i\right)^{\frac{1}{1-2\beta}} \beta^{\frac{2\beta}{1-2\beta}} (1-2\beta). \tag{47}$$

However, to assess whether or not the laissez-faire equilibrium will indeed fulfil the efficiency conditions (45)-(46), we need to take a closer look at the incentives faced by type-1 agents. In particular, we need to check that type-1 agents would not be better off by replicating the effort choices made by type-2 agents in order to get remunerated according to the productivity of their higher-skilled counterparts. By behaving as mimickers, type-1 agents would obtain a utility given by:

$$\widehat{U}^1 = (e_q^2 e_s^2)^\beta A\theta^2 - p_s e_s^2 - p_q^1 e_q^2. \tag{48}$$

Substituting for e_q^2 and e_s^2 in (48) the values provided, for $i=2$, by (45)-(46) gives:

$$\begin{aligned} \widehat{U}^1 \quad = \quad & \left(p_s p_q^2\right)^{-\frac{\beta}{1-2\beta}} \left(A\theta^2\right)^{\frac{1}{1-2\beta}} \beta^{\frac{2\beta}{1-2\beta}} - (p_s)^{-\frac{\beta}{1-2\beta}} \left(p_q^2\right)^{-\frac{\beta}{1-2\beta}} \left(\beta A\theta^2\right)^{\frac{1}{1-2\beta}} \\ & - (p_s)^{-\frac{\beta}{1-2\beta}} p_q^1 \left(p_q^2\right)^{-\frac{1-\beta}{1-2\beta}} \left(\beta A\theta^2\right)^{\frac{1}{1-2\beta}}. \end{aligned} \tag{49}$$

Thus, a type-1 agent will have no incentive to mimic a type 2 agent, and the laissez-faire equilibrium will be efficient, provided that $U^1 \geq \widehat{U}^1$, an inequality that can be rewritten (using (47), for $i=1$, and (49)) as

$$\left(\theta^1/\theta^2\right)^{\frac{1}{1-2\beta}} \geq \frac{1-\beta-\frac{p_q^1}{p_q^2}\beta}{1-2\beta} \left(p_q^1/p_q^2\right)^{\frac{\beta}{1-2\beta}}. \tag{50}$$

A sufficient condition for (50) to be satisfied is that $1-\beta-\left(p_q^1/p_q^2\right)\beta \leq 0$, i.e. $p_q^1/p_q^2 \geq (1-\beta)/\beta$, highlighting that efficiency under laissez-faire requires that the difference in the cost of acquiring the quality signal is sufficiently large.

4.2 The income tax regime

Exploiting assumption (44) allows us to obtain closed-form expressions for the incentive constraints (35)-(36). To achieve this goal, we begin by deriving the effort cost sustained by agents who choose the point on the income tax schedule intended for them.

Choices of a truthfully reporting agent Consider type i agents who earn the income level y^i intended for them by the government. They will choose an efficient mix of e_s and e_q, solving:

$$\min_{e_s,e_q} p_s e_s + p_q^i e_q \quad \text{subject to} \quad (e_s e_q)^\beta A\theta^i = y^i. \tag{51}$$

The optimal effort-choices are given by

$$e_s^i\left(y^i\right) = \sqrt{\left(\frac{y^i}{A\theta^i}\right)^{1/\beta}\frac{p_q^i}{p_s}} \quad \text{and} \quad e_q^i\left(y^i\right) = \sqrt{\left(\frac{y^i}{A\theta^i}\right)^{1/\beta}\frac{p_s}{p_q^i}}. \tag{52}$$

Insertion of (52) into the cost function yields:

$$R^i(y^i) = p_s\sqrt{\left(\frac{y^i}{A\theta^i}\right)^{1/\beta}\frac{p_q^i}{p_s}} + p_q^i\sqrt{\left(\frac{y^i}{A\theta^i}\right)^{1/\beta}\frac{p_s}{p_q^i}} = 2\sqrt{\left(\frac{y^i}{A\theta^i}\right)^{1/\beta}p_s p_q^i}, \quad i = 1, 2. \tag{53}$$

Optimal deviating strategies Consider now deviating strategies. There are three cases to consider depending on which of the two constraints (35)-(36) that are relevant.[36] These cases can be distinguished using conditions that depend on the ratio θ^2/θ^1, the relative size of the two groups (γ^1 and γ^2), and a constant defined as:

$$\Omega \equiv \left[\left(p_q^2 + p_q^1\right) / \left(2\sqrt{p_q^2 p_q^1}\right)\right]^{2\beta}. \tag{54}$$

Case 1: $\theta^2/\theta^1 \leq \Omega$ In this case we have that, in the constraint (35), $\min\left\{\widetilde{R}^2(y^1), \hat{R}^2(y^1)\right\} = \widetilde{R}^2(y^1)$, and in the constraint (36), $\min\left\{\breve{R}^1(y^2), \widetilde{R}^1(y^2)\right\} = \widetilde{R}^1(y^2)$. The effort mix chosen by a type-i mimicker under his/her optimal deviating strategy is undistorted (satisfies $e_q/e_s = p_s/p_q^i$). Moreover, $\widetilde{R}^2(y^1) = 2\sqrt{\left[y^1/\left(A\theta^2\right)\right]^{1/\beta} p_q^2 p_s}$ and $\widetilde{R}^1(y^2) = 2\sqrt{\left[y^2/\left(A\theta^1\right)\right]^{1/\beta} p_q^1 p_s}$. Thus, the relevant *downward* IC constraint can be expressed as:

$$c^2 - R^2(y^2) \geq c^1 - 2\sqrt{\left[y^1/\left(A\theta^2\right)\right]^{1/\beta} p_q^2 p_s}, \tag{55}$$

whereas the *upward* IC-constraint will be necessarily slack.

[36] In our numerical example, we will vary parameters in such a way that all three cases are considered. The derivations needed to distinguish between the different cases are available upon request.

Case 2: $\gamma^1 + (\theta^2/\theta^1)\,\gamma^2 < \Omega < \theta^2/\theta^1$ In this case we have again that, in the constraint (35), $\min\left\{\widetilde{R}^2(y^1), \hat{R}^2(y^1)\right\} = \widetilde{R}^2(y^1)$. This time, however, type-2 mimickers needs to select a distorted effort mix ($e_q/e_s \neq p_s/p_q^2$) in order to achieve separation and be paid according to their true productivity. For the constraint (36) we have instead that $\min\left\{\breve{R}^1(y^2), \widetilde{R}^1(y^2)\right\} = \breve{R}^1(y^2)$. Thus, the relevant *downward* IC constraint can be formulated as:

$$c^2 - R^2(y^2) \geq c^1 - \sqrt{p_s p_q^1}\frac{\sqrt{\left(y^1/A\right)^{1/\beta}}\left(1/\theta^2\right)^{1/\beta}}{\sqrt{\left(1/\theta^1\right)^{1/\beta}} + \sqrt{\left(1/\theta^1\right)^{1/\beta} - \left(1/\theta^2\right)^{1/\beta}}}$$
$$- p_q^2\sqrt{p_s/p_q^1}\sqrt{\left(y^1/A\right)^{1/\beta}}\left[\sqrt{\left(1/\theta^1\right)^{1/\beta}} + \sqrt{\left(1/\theta^1\right)^{1/\beta} - \left(1/\theta^2\right)^{1/\beta}}\right], \tag{56}$$

and the relevant *upward* IC constraint as:

$$c^1 - R^1(y^1) \geq c^2 - \sqrt{\left[y^2/\left(A\theta^2\right)\right]^{1/\beta} p_s/p_q^2}\left(p_q^2 + p_q^1\right). \tag{57}$$

Case 3: $\gamma^1 + (\theta^2/\theta^1)\,\gamma^2 \geq \Omega$ In this case it is not possible to unambiguously determine whether, in the constraint (35), $\widetilde{R}^2(y^1) < \hat{R}^2(y^1)$ or vice versa. What can be established is that the mimicking strategy with associated cost $\widetilde{R}^2(y^1)$ necessarily requires that a type-2 mimicker selects a distorted effort mix. Thus, there will be two relevant *downward* IC constraints, one provided by (56) (the one associated with cost $\widetilde{R}^2(y^1)$), and the other (associated with the cost $\hat{R}^2(y^1)$) given by:

$$c^2 - R^2(y^2) \geq c^1 - 2\sqrt{\left(y^1/A\bar{\theta}\right)^{1/\beta} p_s p_q^2}. \tag{58}$$

For the constraint (36) we have again that $\min\left\{\breve{R}^1(y^2), \widetilde{R}^1(y^2)\right\} = \breve{R}^1(y^2)$. Once more, therefore, the relevant *upward* constraint is given by (57).

4.3 Government problem, extended tax regime

In the extended tax regime, obtaining closed form solutions to the incentive constraints using the functional form assumption in (44) is more straightforward. We begin with the separating equilibrium. In this case, equations (42) and (43) take the form:

$$e_q^{i*} = \left(\frac{y^i}{A\theta^i}\right)^{1/\beta}\frac{1}{e_s^i}, (i = 1, 2) \quad \text{and} \quad \hat{e}_q = \left(\frac{y^1}{A\bar{\theta}}\right)^{1/\beta}\frac{1}{e_s^1}. \tag{59}$$

Thus, the incentive constraints (40)-(41) can be written as follows:

$$c^1 - p_s^1 e_s^1 - \left(\frac{y^1}{A\theta^1}\right)^{1/\beta}\frac{p_q^1}{e_s^1} \geq c^2 - p_s^1 e_s^2 - \left(\frac{y^2}{A\theta^2}\right)^{1/\beta}\frac{p_q^1}{e_s^2}, \tag{60}$$

$$c^2 - p_s^2 e_s^2 - \left(\frac{y^2}{A\theta^2}\right)^{1/\beta}\frac{p_q^2}{e_s^2} \geq c^1 - p_s^2 e_s^1 - \left(\frac{y^1}{A\bar{\theta}}\right)^{1/\beta}\frac{p_q^2}{e_s^1}. \tag{61}$$

When implementing a pooling equilibrium, incentive constraints can be neglected, and, in accordance with (37), the government chooses (y, e_s) to maximize

$$u^1 = y - p_s^1 e_s + p_q^1 \widehat{e}_q(y, e_s), \tag{62}$$

where $\widehat{e}_q(e_s, y)$ is the value of e_q which solves the equation $y = (e_s e_q)^\beta A\bar{\theta}$.

4.4 Numerical results

We fix the productivity of type 2 to $\theta^2 = 100$ and compute the social welfare level, distortions, and income levels in the various tax regimes when θ^1 varies between 1 and 100. In this way, we consider a wide range of values for the ratio θ^1/θ^2. We maintain our previous normalization $p_s^1 = p_s^2 = p_q^2 = 1$ and set $\beta = 0.25$, $A = 1$, $\gamma^1 = \gamma^2 = 0.5$. We consider two scenarios regarding the difference in the cost of acquiring e_q between the two types of agents. In the first scenario, we consider a small value of $p_q^1 - p_q^2$, letting $p_q^1 = 1.05$. In the second scenario we consider a large value of $p_q^1 - p_q^2$, letting $p_q^1 = 1.5$.[37]

Since our model is quite stylized, we refrain from making a precise empirical calibration. However, in principle one could interpret $\theta^2 - \theta^1$ as the difference in labor productivity between college and non-college educated workers. This would suggest that values of θ^1 between 66 and 85 would be realistic given that studies have found college wage premiums that range between 18 and 50 per cent.[38] Regarding $p_q^1 - p_q^2$, this could represent differences in the costs of acquiring education depending on parental background (capturing, for example, whether the parents are college-educated or not).

In our simulations, we report the maximum achievable welfare gain. This maximum welfare gain is computed at the value of θ^1 where the difference between the social welfare level in the extended regime and the social welfare level in the income tax regime is the largest, and we focus on an equivalent-variation type of welfare measure.[39]

Figure 2 shows the results for the case when the difference $p_q^1 - p_q^2$ is small. As expected, we see from the top left panel, that the extended tax regime always welfare-dominates the income tax regime. More interestingly, we see that the it is optimal to implement a separating allocation when θ^1 takes on low and intermediate values, whereas the pooling allocation dominates when

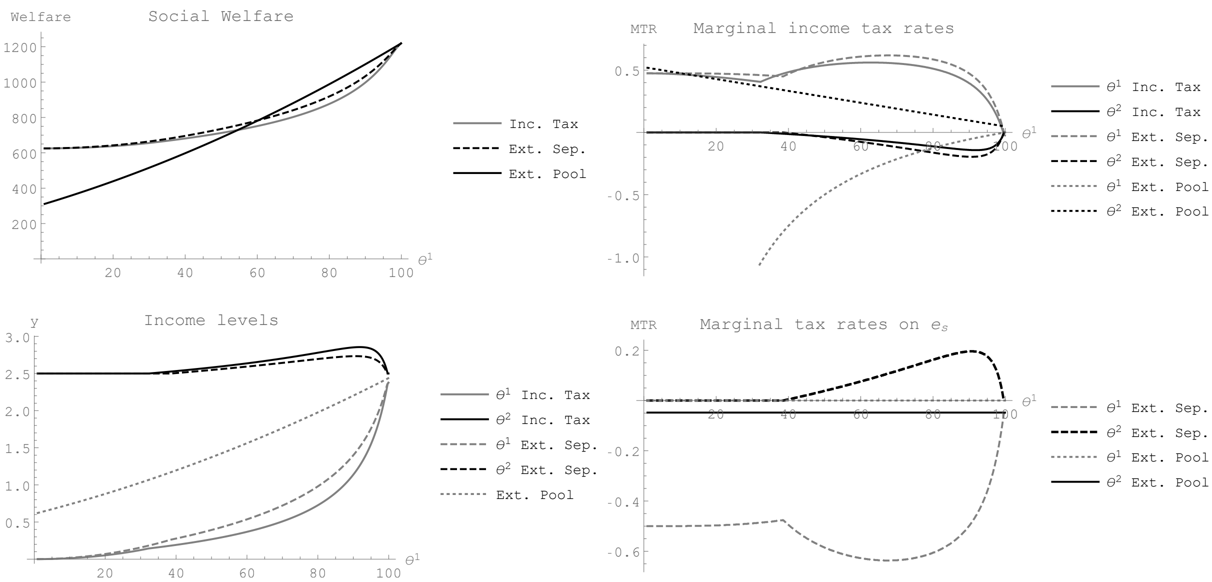
[37] The qualitative features of the results are robust to changes in p_q^1 and β.

[38] See e.g., van der Velden and Bijlsma (2016) who use recent data from 22 OECD countries and find that the college wage premium ranges between 18 percent (Sweden) and 50 per cent (Slovak Republic). Ashworth and Ransom (2019) find that the college wage premium in the US ranges between 40 and 50 per cent depending on cohort and data set used. The numbers in the text are based on the calculation $100/85 \approx 1.1765$ and $100/66 \approx 1.5152$.

[39] The equivalent-variation type welfare gain is obtained by first calculating the wage level θ^1 at which the vertical distance between the social welfare levels of the extended tax regime and the income tax regime is the largest. We then compute the minimal amount of resources that must be injected into the income tax regime in order to reach the social welfare level of the extended tax regime (repeatedly solving the government optimization program). Finally, we divide this number by the aggregate output of the income tax regime to get a welfare gain measure expressed as a fraction of output.

θ^1 is relatively close to θ^2.[40] The maximum welfare gain of taxing the signal is obtained at $\theta^1 = 84.1$, amounts to 6.31% of aggregate output, and is associated with the implementation of a pooling allocation.

Figure 2: Numerical illustration, small difference in $p_q^1 - p_q^2$



Turning to the top right panel of figure 2, we see that the income tax regime and the extended separating regime always imply a positive marginal tax rate on low-skilled agents. This is a standard property in optimal income tax models and serves to mitigate the binding downwards IC constraint. The negative marginal tax rate is non-standard and is due to the binding upwards incentive constraint. It can also be noted that the earned income distortions in the separating extended regime are more pronounced than in the income tax regime. In the extended pooling regime, a positive marginal tax rate is levied on high-skill agents and a negative marginal tax rate is levied on low-skill agents. This is due to the cross-subsidization between the two types in the pooling allocation. Moreover, since the degree of cross-subsidization decreases when θ^1 approaches θ^2, the marginal tax rates converge when θ^1 approaches θ^2.

The bottom-left panel shows the income levels in the different optimal tax regimes. Not surprisingly, the income levels associated with the pooling regime lie in between the income levels of the tax regimes where the labor market equilibrium is separating. Moreover, the income levels of the two agents converge as the two agents become more similar in terms of θ.

Finally, we turn to the bottom-right panel. Notice that, in this panel, we are only representing the graphs of the extended tax regime. The reason is that, by construction, the mix of effort inputs is always undistorted with only an income tax in place. In the separating extended tax regime, we note that it is always desirable to subsidize the observable dimension of education effort e_s of type-1 agents (reflected in an upwards-distortion). The reason is that subsidizing the

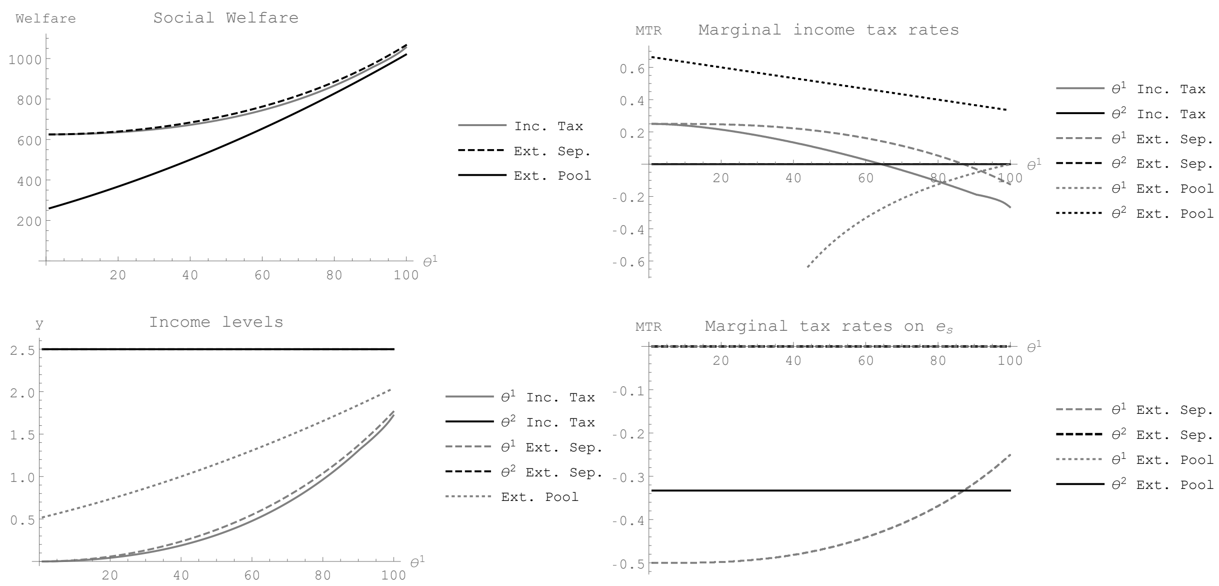
[40]Notice that when θ^1 is very close to 100, the separating allocation dominates, even though it is not visible in the figure. This is a knife-edge case of no practical relevance.

dimension of education effort in which low-skill agents have a comparative advantage, mitigates the incentive constraint of the high-skill type. For the high-skill type, it is desirable to tax e_s over the range of values of θ^1 for which the upwards incentive constraint is binding. This can be understood from the fact that by taxing the quantity dimension, we are implicitly subsidizing the *quality* dimension in which the high-skill type has a comparative advantage. This serves to mitigate the binding incentive constraint of the low-skill type. In the pooling regime, there is by definition no binding incentive constraint. As we are invoking a max-min welfare function, the optimum is simply to achieve the best effort mix from the perspective of the low skilled type. This implies an efficient effort mix for the low skilled type and, hence, an upwards distortion on e_s for the high-skilled agents, due to their comparative advantage in the quality dimension of effort.

The numerical example illustrates that both education subsidies and taxes are warranted on redistributive grounds to mitigate binding IC constraints. Notably, and contrary to conventional wisdom, a tax on education may be desirable. It may be required to implement a separating allocation, via mitigating the incentives of the low-skilled workers to mimic their higher-skilled counterparts, when employers are unable to observe the true productivity of their employees.

Figure 3 shows the results for the case when the difference $p_q^1 - p_q^2$ is relatively large. Again, as expected, the extended tax regime always welfare-dominates the income tax regime. However, in contrast to the case when $p_q^1 - p_q^2$ is relatively small, we can see that pooling is always suboptimal. This reflects the fact that, as the difference in the costs of acquiring the signals becomes larger and larger, it becomes less likely that an optimal separating equilibrium features a double distortion with both a downward and an upward binding IC constraint. In particular, when the difference in the costs of acquiring the signals is sufficiently large, the optimal separating equilibrium will preserve efficiency at the top (and only entail a distortion on the behavior of low-skilled agents). In Figure 3 this is shown by the fact that both in the top-right- and bottom-right panel, the black dashed line coincides with the horizontal axis (in contrast to what happens in Figure 2). The maximum welfare gain of taxing the signal is obtained at $\theta^1 = 71.0$, amounts to 1.20% of aggregate output, and is associated with a separating allocation.

Figure 3: Numerical illustration, large difference in $p_q^1 - p_q^2$



As we highlighted in Section 3.3, where we compared the relative merits of pooling versus separating equilibria, the attractiveness of implementing a pooling equilibrium hinges on its equity gains, i.e. the fact that it fully eliminates all the information rent (derived by high-skilled agents) associated with the difference in productivities. Obviously, this gain is bigger the larger the information rent, associated with differences in productivities, that is enjoyed by high-skilled agents under an optimal separating equilibrium. On the other hand, whether this equity gain of pooling can be reaped cheaply or not (in efficiency terms), depends on the magnitude of the difference in the costs of acquiring the signals (in our case, the magnitude of the difference $p_q^1 - p_q^2$). This is due to the fact that an optimal pooling equilibrium always violates efficiency at the top. Instead, an optimal separating equilibrium preserves efficiency at the top when the difference $p_q^1 - p_q^2$ is sufficiently large.

5 Conclusions

We have analyzed optimal redistribution in the presence of signaling, introducing two realistic new features to the standard Mirrleesian framework: (i) the existence of a second layer of asymmetric information between employers and workers regarding the productivity of workers, with the latter having the possibility of engaging in signaling to credibly convey this information to prospective employers; (ii) an extended tax system which conditions taxes/transfers on the income earned by workers as well as on the signal(s) acquired by them. The combination of these two new features has been shown to maintain the second-best nature of the government optimization problem and the inherent trade-off between conflicting equity and efficiency considerations.

We have focused on a model with two types of agents (low- and high-ability) and considered both a setting with uni-dimensional signaling, and one with bi-dimensional signaling in which the high-skilled exhibit a comparative advantage relative to the lower-skilled counterparts. To ensure the possibility that agents credibly signal their true type to prospective employers, we have assumed that low and high-skilled agents differ in the cost of acquiring the signal(s).

We have shown that the socially optimal tax equilibrium could either be given by a separating or by a pooling allocation, demonstrating how the government can employ the extended tax system to either enhance the efficiency of redistribution through the income channel (under the former configuration), or, to induce a pre-distributive change in the wage structure, resulting in cross-subsidization between skill levels (under the latter configuration).

From a policy perspective, viewing educational attainment as a signaling device used by high skilled workers to separate themselves in the labor market from their lower-skilled counterparts, our analysis calls for revisiting commonly applied policy tools, such as education mandates and (means-tested) education subsidies. These policy tools are often warranted on efficiency grounds to address prevalent market failures (e.g., alleviating credit constraints or internalizing externalities). However, we argue that these tools can be viewed as forms of taxes levied on the signals acquired by workers, thereby serving to promote redistributive goals.

Our analysis takes an initial step in exploring redistributive policy in the presence of signaling and we invoke several restrictive simplifying assumptions to gain tractability. In particular, we have limited the analysis to a setup with two types of agents. However, our main qualitative insights carry over to the general case with many types. In the general case, the social optimum could be given by a hybrid allocation that combines pre-distribution (bunching) with redistribution, rather than taking one of the two extreme configurations (full separation or pooling) as under the two-type setting. Introducing the possibility to tax signals (on a means-tested basis) may substantially enhance welfare relative to the optimal income tax and transfer system; and, unless one can condition the tax on the entire set of signals, strengthens the case for pre-distribution when a high-type has no comparative advantage within the set of untaxed signals.

An interesting extension of our analysis is with respect to the context of signaling. In our setup, workers engaged in signaling to convey credible information about their productive ability to prospective employers in order to get a higher remuneration. Pre-distribution hence was carried out via re-distribution through the wage channel. One could alternatively consider signaling via conspicuous consumption that serves to enhance social status, in which case pre-distribution would take the form of redistributing via the status channel. We leave this as a topic for future research.

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A Proof of Proposition 1

Consider the government's problem presented in the second part of Section 2.2.1. Denote by δ^i, for $i = 1, 2$, the Lagrange multiplier associated with the constraint (9), by λ^1 the Lagrange multiplier associated with the constraint (10), by λ^2 the Lagrange multiplier associated with the constraint (11), and by μ the Lagrange multiplier associated with the constraint (12). The first order conditions (with respect to, respectively, y^2, e^2, c^2, y^1, e^1, c^1) are :

$$\delta^2 = \mu\gamma^2, \tag{A 1}$$

$$\lambda^2 g'\left(e^2\right) = \delta^2\theta^2 + \lambda^1 k g'\left(e^2\right), \tag{A 2}$$

$$\lambda^2 = \mu\gamma^2 + \lambda^1, \tag{A 3}$$

$$\delta^1 = \mu\gamma^1 + \frac{\lambda^2 g'\left(y^1/\overline{\theta}\right)}{\overline{\theta}}, \tag{A 4}$$

$$\delta^1\theta^1 - \left(1+\lambda^1\right) k g'\left(e^1\right) = 0, \tag{A 5}$$

$$1 + \lambda^1 - \lambda^2 = \mu\gamma^1. \tag{A 6}$$

Substituting for λ^2 in (A 6) the RHS of (A 3) gives $1 + \lambda^1 - \mu\gamma^2 - \lambda^1 = \mu\gamma^1$, from which one can conclude that $\mu = 1$. Exploiting this result, one can use (A 1) to rewrite (A 2) as

$$\lambda^2 g'\left(e^2\right) = \gamma^2\theta^2 + \lambda^1 k g'\left(e^2\right). \tag{A 7}$$

Dividing (A 7) by (A 3), and multiplying both sides of the resulting equation by the RHS of (A 3), gives (taking into account that $\mu = 1$)

$$\left(\gamma^2 + \lambda^1\right) g'\left(e^2\right) = \gamma^2\theta^2 + \lambda^1 k g'\left(e^2\right),$$

or, equivalently:

$$\gamma^2\theta^2\left[1 - \frac{g'\left(e^2\right)}{\theta^2}\right] = -\left(k-1\right)\lambda^1 g'\left(e^2\right).$$

Finally, dividing both sides of the equation by $\gamma^2\theta^2$ enables to establish (14) in Proposition 1:

$$1 - \frac{g'\left(e^2\right)}{\theta^2} = -\frac{\lambda^1}{\gamma^2}\frac{g'\left(e^2\right)}{\theta^2}\left(k-1\right).$$

Substituting for δ^1 in (A 5) the RHS of (A 4), and exploiting the fact that $\mu = 1$, allows re-expressing (A 5) as follows:

$$\left(1+\lambda^1\right) kg'\left(e^1\right) = \gamma^1\theta^1 + \frac{\lambda^2\theta^1 g'\left(y^1/\overline{\theta}\right)}{\overline{\theta}}. \tag{A 8}$$

Rewrite (A 6) as

$$1+\lambda^1 = \lambda^2 + \gamma^1, \tag{A 9}$$

where we have taken into account that $\mu = 1$.

Dividing (A 8) by (A 9), and multiplying both sides of the resulting equation by the RHS of (A 9), gives

$$\left(\lambda^2+\gamma^1\right) kg'\left(e^1\right) = \gamma^1\theta^1 + \frac{\lambda^2\theta^1 g'\left(y^1/\overline{\theta}\right)}{\overline{\theta}}, \tag{A 10}$$

which can be equivalently rewritten as

$$\left(1-\frac{kg'\left(e^1\right)}{\theta^1}\right)\gamma^1\theta^1 = \lambda^2\left[kg'\left(e^1\right) - \frac{\theta^1 g'\left(y^1/\overline{\theta}\right)}{\overline{\theta}}\right], \tag{A 11}$$

from which we obtain:

$$1-\frac{kg'\left(y^1/\theta^1\right)}{\theta^1} = \frac{\lambda^2}{\gamma^1}\left[\frac{kg'\left(y^1/\theta^1\right)}{\theta^1} - \frac{g'\left(y^1/\overline{\theta}\right)}{\overline{\theta}}\right]. \tag{A 12}$$

Since from (A 3) we have that $\lambda^2 = \mu\gamma^2 + \lambda^1 = \gamma^2 + \lambda^1$, we can equivalently rewrite (A 12) as

$$1-\frac{kg'\left(y^1/\theta^1\right)}{\theta^1} = \frac{\gamma^2+\lambda^1}{\gamma^1}\frac{g'\left(y^1/\theta^1\right)}{\theta^1}\left[k - \frac{\theta^1}{\overline{\theta}}\frac{g'\left(y^1/\overline{\theta}\right)}{g'\left(y^1/\theta^1\right)}\right],$$

which is equal to (13) in Proposition 1.

B Proof of Proposition 3

Part (i) Suppose by negation that for some $\theta^2 > \theta^1 > 0$ the social optimum is given by a pooling allocation, which satisfies, $\hat{c} = \hat{y} = \hat{e}\cdot\sum_i \gamma^i\theta^i$. To show that pooling is suboptimal, consider the following reform starting from an initial pooling equilibrium:

$$\widetilde{e}^1 = \widehat{e} - \epsilon, \quad \widetilde{c}^1 = \widehat{c} - \epsilon kg'\left(\widehat{e}\right), \tag{B 1}$$

$$\widetilde{e}^2 = \widehat{e} + \delta, \quad \widetilde{c}^2 = \widehat{c} + \delta g'\left(\widehat{e}\right), \tag{B 2}$$

where $\epsilon > 0$ and $\delta > 0$. By construction, the reform is welfare neutral for both types of workers since we have that $dU^1 = dc - (kg')\,de = -\epsilon kg'\left(\widehat{e}\right) + \epsilon kg'\left(\widehat{e}\right) = 0$ and $dU^2 = dc - (g')\,de = \delta g'\left(\widehat{e}\right) - \delta g'\left(\widehat{e}\right) = 0$. Notice also that, after the reform is implemented, no agent is tempted to behave as a mimicker. By behaving as a mimicker, the utility of a type-1 agent would change by $\delta g'(\widehat{e}) - \delta k g'(\widehat{e}) < 0$ (since $k > 1$). Similarly, by behaving as a mimicker, the utility of a type-2 agent would change by $-\epsilon k g'(\widehat{e}) + \epsilon g'(\widehat{e}) < 0$ (since $k > 1$). Let's now consider the effect of the proposed reform on the net revenue collected by the government. We know that net revenue is 0 in the initial equilibrium. Net revenue after the reform is equal to the difference between the variation in aggregate output and the variation in aggregate consumption:

$$dY - dC = \underbrace{\left[\gamma^2\delta\theta^2 - \gamma^1\epsilon\theta^1\right]}_{dY} - \underbrace{\left[\gamma^2\delta - \gamma^1\epsilon k\right] g'(\widehat{e})}_{dC} = \gamma^2\delta\left[\theta^2 - g'(\widehat{e})\right] - \gamma^1\epsilon\left[\theta^1 - k g'(\widehat{e})\right]. \tag{B 3}$$

Notice that $dY - dC$ will be necessarily positive when at the initial pooling equilibrium we have $\theta^2 - g'(\widehat{e}) > 0$ and $\theta^1 - k g'(\widehat{e}) < 0$. Notice also that it is not possible that at the initial pooling equilibrium we have at the same time that $\theta^2 - g'(\widehat{e}) < 0$ and $\theta^1 - k g'(\widehat{e}) > 0$ (this is due to the fact that $\theta^2 > \theta^1$ and $k > 1$). Suppose then that at the initial pooling equilibrium we have that $\theta^2 - g'(\widehat{e}) > 0$ and $\theta^1 - k g'(\widehat{e}) > 0$. Thus, net revenue increases whenever $\delta > \epsilon \frac{\gamma^1}{\gamma^2} \frac{\theta^1 - k g'(\widehat{e})}{\theta^2 - g'(\widehat{e})}$. In other words, there exists a reform to the presumed optimal allocation that yields a fiscal surplus which can be rebated to the workers in a lump-sum fashion without violating the IC constraints (due to the quasi-linearity of preferences). Hence, a Pareto improvement can be obtained. This gives us the desired contradiction.

Part (ii) It suffices to show that the welfare level associated with a separating equilibrium is higher under the extended tax regime than under the pure income tax regime. This follows from the fact that the pooling equilibrium yields the same welfare level under the two tax regimes. Thus, in case the optimal solution under a pure income tax regime calls for implementing a pooling equilibrium, it is clearly dominated by the optimal solution under the extended tax regime, by virtue of part (i) of the proposition. In the optimal separating equilibrium under an income tax regime, the incentive compatibility constraint associated with type-2 workers [given by condition (11)] is binding. Otherwise, by continuity considerations one could transfer units of consumption from type-2 to type-1 workers and enhance the latter's level of utility, without violating the incentive compatibility constraint associated with type-2 workers (and clearly without violating the revenue constraint). Under an extended tax regime, the incentive constraint in (11) is replaced by the weaker constraint given by (18). Thus, the optimal separating equilibrium under the pure income tax regime satisfies the reformulated incentive constraint [given in (18)] as a strict inequality.[41] This slack allows the government to enhance the utility of type-1 by slightly increasing the income tax levied on type-2 while increasing the transfer given to type-1, without violating the IC constraint of type 2, maintaining budget balance (by continuity considerations). This establishes part (ii).

[41] The only (knife-edge) cases in which no slack exists are when either $\theta^1 = 0$, or, $\theta^1 = \theta^2$.

Part (iii) We begin by showing that constraints (17) and (18) cannot be binding at the same time. Assume by contradiction that they are both binding. From (17) we get:

$$c^1 = c^2 + k\left[g(e^1) - g(e^2)\right]. \tag{B 4}$$

Substituting for c^1 in (18) the RHS of the equation above gives:

$$c^2 - g(e^2) = c^2 + k\left[g(e^1) - g(e^2)\right] - g(e^1) \Longleftrightarrow (1-k)\left[g(e^1) - g(e^2)\right] = 0, \tag{B 5}$$

a result that cannot hold under a separating equilibrium (where $e^1 \neq e^2$) given that $k \neq 1$. Having established that constraints (17) and (18) cannot be binding at the same time, it follows that constraint (18) is binding and (17) is slack, given the max-min social welfare function. Denote by δ^i, for $i = 1, 2$, the Lagrange multipliers associated with the constraint (16), by λ the multiplier associated with the constraint (18), and by μ the multiplier associated with the constraint (19). The first order conditions of the government's problem (with respect to, respectively, y^2, e^2, c^2, y^1, e^1, c^1) are given by:

$$\begin{aligned}
\delta^2 &= \mu\gamma^2 && \text{(B 6)}\\
\delta^2\theta^2 - \lambda g'\left(e^2\right) &= 0 && \text{(B 7)}\\
\lambda &= \mu\gamma^2 && \text{(B 8)}\\
\delta^1 &= \mu\gamma^1 && \text{(B 9)}\\
\delta^1\theta^1 - kg'\left(e^1\right) + \lambda g'\left(e^1\right) &= 0 && \text{(B 10)}\\
1 - \lambda &= \mu\gamma^1 && \text{(B 11)}
\end{aligned}$$

Using (B 8) to substitute $\mu\gamma^2$ for λ in (B 11) gives the result that $\mu = 1$. From (B 6) we then obtain that $\delta^2 = \gamma^2$, and from (B 8) we obtain that $\lambda = \gamma^2$. Thus, one can rewrite (B 7) as $\gamma^2\left[\theta^2 - g'\left(e^2\right)\right] = 0$, which implies that $1 = g'\left(e^2\right)/\theta^2$, i.e., (21) is satisfied. Exploiting the fact that $\mu = 1$ and therefore, from (B 9), $\delta^1 = \gamma^1$, we can rewrite (B 10) as

$$kg'\left(e^1\right) = \lambda g'\left(e^1\right) + \gamma^1\theta^1. \tag{B 12}$$

Since (B 11) can be equivalently rewritten (taking into account that $\mu = 1$) as $1 = \lambda + \gamma^1$, combining (B 12) with (B 11) allows obtaining that

$$kg'\left(e^1\right)\left[\lambda + \gamma^1\right] = \lambda g'\left(e^1\right) + \gamma^1\theta^1, \tag{B 13}$$

from which one gets (exploiting that $\lambda = \gamma^2$):

$$1 - \frac{kg'\left(e^1\right)}{\theta^1} = \frac{\gamma^2}{\gamma^1}\frac{g'\left(e^1\right)}{\theta^1}\left(k-1\right). \tag{B 14}$$

The RHS of (B 14) provides a measure of the distortion imposed on type-1 agents for self-selection reasons and is strictly positive, implying that the effort provided by type-1 agents is downward distorted. Since from (B 12) we obtain (taking into account that $\lambda = \gamma^2$) that $g'(e^1)/\theta^1 = \gamma^1/(k-\gamma^2)$, we can restate (B 14) as (20).

To show that the distortion on type 1 in the extended tax regime is strictly smaller than in the income tax regime, denote by e^1_{IT} the value of e^1 at separating tax-equilibrium with only an income tax in place, and by e^1_{EXT} the value of e^1 at a separating tax-equilibrium under an extended tax regime. We have that

$$1 - \frac{kg'(e^1_{IT})}{\theta^1} = \frac{\gamma^2+\lambda^1}{\gamma^1}\frac{g'(e^1_{IT})}{\theta^1}\left[k - \frac{\theta^1}{\overline{\theta}}\frac{g'\left(e^1_{IT}\theta^1/\overline{\theta}\right)}{g'(e^1_{IT})}\right], \tag{B 15}$$

and

$$1 - \frac{kg'(e^1_{EXT})}{\theta^1} = \frac{\gamma^2}{\gamma^1}\frac{g'(e^1_{EXT})}{\theta^1}(k-1). \tag{B 16}$$

Suppose first that $\lambda^1 = 0$ in (B 15). Given that $\frac{\theta^1}{\overline{\theta}}\frac{g'\left(e^1_{IT}\theta^1/\overline{\theta}\right)}{g'\left(e^1_{IT}\right)} > 1$, if we were to plug the value e^1_{IT} into (B 16) we would obtain that $1 - \frac{kg'\left(e^1_{EXT}\right)}{\theta^1} > \frac{\gamma^2}{\gamma^1}\frac{g'\left(e^1_{EXT}\right)}{\theta^1}(k-1)$. Thus, it must be that $e^1_{EXT} > e^1_{IT}$. Now suppose that $\lambda^1 > 0$ in (B 15). In this case e^1_{IT} is more downward distorted than when $\lambda^1 = 0$, whereas the value of e^1_{EXT} does not change (given that λ^1 does not enter into (B 16). Thus, the conclusion that $e^1_{EXT} > e^1_{IT}$ still applies.

C Proof of Proposition 8

Part (i) The first order conditions of the government's problem (37) can be written:

$$1 - p^1_q \frac{\partial \widehat{e}_q}{\partial y} = 0, \tag{C 1}$$

$$-p_s + p^1_q \frac{\partial h(e_s, \widehat{e}_q)/\partial e_s}{\partial h(e_s, \widehat{e}_q)/\partial e_q} = 0. \tag{C 2}$$

Eq. (C 2) shows that, at the optimal pooling equilibrium, type-1 agents choose an undistorted effort mix and implies, given that $p^1_q > p^2_q$, that type-2 agents are forced to choose a distorted effort mix, with $e^2_q = \widehat{e}_q$ being distorted downwards. As (C 1) equivalently can be written as $1 - \frac{p^1_q}{\overline{\theta}\partial h(e_s,e_q)/\partial e_q} = 0$, and since an undistorted level of income would satisfy the condition $1 - \frac{p^1_q}{\theta^1 \partial h(e_s,e_q)/\partial e_q} = 0$ for type-1 agents and $1 - \frac{p^2_q}{\theta^2 \partial h(e_s,e_q)/\partial e_q} = 0$ for type-2 agents, it also follows that at the optimal pooling equilibrium y is distorted upwards for type-1 agents and downwards for type-2 agents.

Part ii) Below we show that an optimal separating equilibrium the following conditions hold:

$$p_s - p_q^1 \frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} \;>\; 0, \tag{C 3}$$

$$p_s - p_q^2 \frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} \;\leq\; 0. \tag{C 4}$$

According to inequality (C 3), the effort mix chosen by type-1 agents is distorted towards e_s, whereas according to (C 4) the effort mix chosen by type-2 agents is either left undistorted (which happens when the constraint (41) is slack) or is distorted towards e_q (when the constraint (41) is binding). We also show below that an optimal separating equilibrium satisfies $1 - \frac{p_q^1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} > 0$ and $1 - \frac{p_q^2}{\theta^2 \partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} \leq 0$, implying that y^1 is distorted downwards (to deter mimicking by type-2 agents) and y^2 is either left undistorted (when the constraint (41) is slack) or is distorted upwards (when the constraint (41) is binding).

Consider the problem solved by a government under an extended tax regime and denote μ the Lagrange multiplier attached to the constraint (39), by λ^2 the multiplier attached to the constraint (40) and by λ^1 the multiplier attached to the constraint (41). The first order conditions for y^1, e_s^1, c^1, y^2, e_s^2, c^2 are respectively given by:

$$\left(1+\lambda^1\right) \frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1, e_s^1, \theta^1\right)} \frac{\partial e_q^{1*}}{\partial y^1} = \lambda^2 \frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1, e_s^1, \overline{\theta}\right)} \frac{\partial \hat{e}_q^2}{\partial y^1} + \mu\gamma^1, \tag{C 5}$$

$$\begin{aligned} &\left(1+\lambda^1\right)\left[\frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_s^1} + \frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1, e_s^1, \theta^1\right)} \frac{\partial e_q^{1*}}{\partial e_s^1}\right] \\ = \;& \lambda^2\left[\frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial e_s^1} + \frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1, e_s^1, \overline{\theta}\right)} \frac{\partial \hat{e}_q^2}{\partial e_s^1}\right], \end{aligned} \tag{C 6}$$

$$1+\lambda^1 = \lambda^2 + \mu\gamma^1, \tag{C 7}$$

$$\lambda^2 \frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial y^2} = \lambda^1 \frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial y^2} + \mu\gamma^2, \tag{C 8}$$

$$\begin{aligned} &\lambda^2\left[\frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_s^2} + \frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial e_s^2}\right] \\ = \;& \lambda^1\left[\frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_s^2} + \frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial e_s^2}\right], \end{aligned} \tag{C 9}$$

$$\lambda^2 = \lambda^1 + \mu\gamma^2. \tag{C 10}$$

Dividing (C 6) by (C 7), and multiplying both sides by the RHS of (C 7) gives:

$$\begin{aligned} & \left[\frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_s^1} + \frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1, e_s^1, \theta^1\right)} \frac{\partial e_q^{1*}}{\partial e_s^1}\right]\left(\lambda^2 + \mu\gamma^1\right) \\ = \; & \lambda^2 \left[\frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial e_s^1} + \frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1, e_s^1, \overline{\theta}\right)} \frac{\partial \hat{e}_q^2}{\partial e_s^1}\right], \end{aligned} \tag{C 11}$$

which can be equivalently rewritten as

$$\left[p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1}\right]\left(\lambda^2 + \mu\gamma^1\right) = \lambda^2 \left[p_s + p_q^2 \frac{\partial \hat{e}_q^2}{\partial e_s^1}\right],$$

from which one obtains

$$p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1} = \frac{\lambda^2}{\mu\gamma^1}\left[\left(p_s + p_q^2 \frac{\partial \hat{e}_q^2}{\partial e_s^1}\right) - \left(p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1}\right)\right], \tag{C 12}$$

and therefore, simplifying terms,

$$p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1} = \frac{\lambda^2}{\mu\gamma^1}\left[p_q^2 \frac{\partial \hat{e}_q^2}{\partial e_s^1} - p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1}\right]. \tag{C 13}$$

Notice that we have

$$\frac{\partial e_q^{1*}}{\partial e_s^1} = -\frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} \quad \text{and} \quad \frac{\partial \hat{e}_q^2}{\partial e_s^1} = -\frac{\partial h\left(e_s^1, \hat{e}_q^2\right)/\partial e_s}{\partial h\left(e_s^1, \hat{e}_q^2\right)/\partial e_q}. \tag{C 14}$$

Moreover, since $e_q^{1*} = e_q^{1*}\left(y^1, e_s^1, \theta^1\right)$ and $\hat{e}_q^2 = \left(y^1, e_s^1, \overline{\theta}\right)$, it follows that $e_q^{1*} > \hat{e}_q^2 > 0$ and therefore $\frac{\partial e_q^{1*}}{\partial e_s^1} < \frac{\partial \hat{e}_q^2}{\partial e_s^1} < 0$. Thus, the RHS of (C 13) is positive (given that our max-min objective implies that $\lambda^2 > 0$). Since $p_s + p_q^1 \frac{\partial e_q^{1*}}{\partial e_s^1} = p_s - p_q^1 \frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q}$, we can conclude that at an optimal separating equilibrium

$$p_s - p_q^1 \frac{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_s}{\partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} > 0. \tag{C 15}$$

Dividing (C 9) by (C 10), and multiplying both sides by the RHS of (C 10) gives:

$$\begin{aligned} & \left[\frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_s^2} + \frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial e_s^2}\right]\left(\lambda^1 + \mu\gamma^2\right) \\ = \; & \lambda^1 \left[\frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_s^2} + \frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial e_s^2}\right], \end{aligned} \tag{C 16}$$

which can be equivalently rewritten as

$$\left[p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2}\right] \left(\lambda^1 + \mu\gamma^2\right) = \lambda^1 \left[p_s + p_q^1 \frac{\partial e_q^{2*}}{\partial e_s^2}\right], \tag{C 17}$$

from which one obtains

$$p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2} = \frac{\lambda^1}{\mu\gamma^2} \left[\left(p_s + p_q^1 \frac{\partial e_q^{2*}}{\partial e_s^2}\right) - \left(p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2}\right)\right], \tag{C 18}$$

and therefore, simplifying terms:

$$p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2} = \frac{\lambda^1}{\mu\gamma^2} \frac{\partial e_q^{2*}}{\partial e_s^2} \left(p_q^1 - p_q^2\right). \tag{C 19}$$

Since we have that $p_q^1 - p_q^2 > 0$ and

$$\frac{\partial e_q^{2*}}{\partial e_s^2} = -\frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} < 0, \tag{C 20}$$

it follows that the RHS of (C 19) is either negative (when $\lambda^1 > 0$) or zero (when $\lambda^1 = 0$). Since $p_s + p_q^2 \frac{\partial e_q^{2*}}{\partial e_s^2} = p_s - p_q^2 \frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q}$, we can conclude that at an optimal separating equilibrium

$$p_s - p_q^2 \frac{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_s}{\partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} \leq 0. \tag{C 21}$$

Dividing (C 5) by (C 7), and multiplying both sides by the RHS of (C 7) gives:

$$\frac{\partial R^1\left(e_s^1, e_q^{1*}\right)}{\partial e_q^{1*}\left(y^1, e_s^1, \theta^1\right)} \frac{\partial e_q^{1*}}{\partial y^1} \left(\lambda^2 + \mu\gamma^1\right) = \lambda^2 \frac{\partial R^2\left(e_s^1, \hat{e}_q^2\right)}{\partial \hat{e}_q^2\left(y^1, e_s^1, \overline{\theta}\right)} \frac{\partial \hat{e}_q^2}{\partial y^1} + \mu\gamma^1, \tag{C 22}$$

which can be equivalently rewritten as

$$p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} \left(\lambda^2 + \mu\gamma^1\right) = \lambda^2 p_q^2 \frac{\partial \hat{e}_q^2}{\partial y^1} + \mu\gamma^1, \tag{C 23}$$

from which one obtains

$$1 - p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} = \frac{\lambda^2}{\mu\gamma^1} \left[p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} - p_q^2 \frac{\partial \hat{e}_q^2}{\partial y^1}\right]. \tag{C 24}$$

Since we have that

$$\frac{\partial e_q^{1*}}{\partial y^1} = \frac{1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} \quad \text{and} \quad \frac{\partial \hat{e}_q^2}{\partial y^1} = \frac{1}{\overline{\theta} \partial h\left(e_s^1, \hat{e}_q^2\right)/\partial e_q}, \tag{C 25}$$

it follows that $\frac{\partial e_q^{1*}}{\partial y^1} > \frac{\partial \hat{e}_q^2}{\partial y^1}$ (taking into account that $\theta^1 < \overline{\theta}$ and $e_q^{1*} > \hat{e}_q^2 > 0$). Thus, the RHS of (C 24) is strictly positive (given that our max-min objective implies that $\lambda^2 > 0$). Since $1 - p_q^1 \frac{\partial e_q^{1*}}{\partial y^1} = 1 - \frac{p_q^1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q}$, it follows that at an optimal separating equilibrium

$$1 - \frac{p_q^1}{\theta^1 \partial h\left(e_s^1, e_q^{1*}\right)/\partial e_q} > 0. \tag{C 26}$$

Dividing (C 8) by (C 10), and multiplying both sides by the RHS of (C 10) gives:

$$\frac{\partial R^2\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial y^2}\left(\lambda^1 + \mu\gamma^2\right) = \lambda^1 \frac{\partial R^1\left(e_s^2, e_q^{2*}\right)}{\partial e_q^{2*}\left(y^2, e_s^2, \theta^2\right)} \frac{\partial e_q^{2*}}{\partial y^2} + \mu\gamma^2, \tag{C 27}$$

which can be equivalently rewritten as

$$p_q^2 \frac{\partial e_q^{2*}}{\partial y^2}\left(\lambda^1 + \mu\gamma^2\right) = \lambda^1 p_q^1 \frac{\partial e_q^{2*}}{\partial y^2} + \mu\gamma^2, \tag{C 28}$$

from which one obtains

$$1 - p_q^2 \frac{\partial e_q^{2*}}{\partial y^2} = \frac{\lambda^1}{\mu\gamma^2} \frac{\partial e_q^{2*}}{\partial y^2}\left(p_q^2 - p_q^1\right). \tag{C 29}$$

Given that $p_q^2 - p_q^1 < 0$, it follows that the RHS of (C 29) is either negative (when $\lambda^1 > 0$) or zero (when $\lambda^1 = 0$). Since $1 - p_q^2 \frac{\partial e_q^{2*}}{\partial y^2} = 1 - \frac{p_q^2}{\theta^2 \partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q}$, it follows that at an optimal separating equilibrium

$$1 - \frac{p_q^2}{\theta^2 \partial h\left(e_s^2, e_q^{2*}\right)/\partial e_q} \leq 0. \tag{C 30}$$

D Condition for full separation, $N > 2$ extended tax regime

Here we provide a heuristic derivation of the necessary and sufficient condition (22). As a preliminary observation, notice that incentive-compatibility requires, $\forall i \in \{1, ..., N-1\}$, that $e^{i+1} \geq e^i$ and $c^{i+1} \geq c^i$; this implies that bunching can only involve adjacent types.[42] Then, suppose to start from an initial equilibrium where two adjacent types, i and $i+1$, are bunched together at a common bundle $(\widehat{y}, \widehat{c}, \widehat{e})$.[43] Consider first whether one can improve upon the initial equilibrium by achieving separation through a reform that raises the effort exerted by type-$i+1$ agents, without affecting their utility and without violating incentive-compatibility. For this purpose, suppose that the government replaces the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ with the bundle $(y^i, \widehat{c}, \widehat{e})$, where $y^i = \theta^i \widehat{e}$, and at the same time offers an additional bundle, which we will denote by $(y^{i+1}, c^{i+1}, e^{i+1})$, such that $e^{i+1} = \widehat{e} + \epsilon$, $c^{i+1} = \widehat{c} + \epsilon k^{i+1} g'\left(\widehat{e}\right)$ and $y^{i+1} = \left(\widehat{e} + \epsilon\right)\theta^{i+1}$, with

[42] The fact that $e^{i+1} \geq e^i$ and $c^{i+1} \geq c^i$ also implies that $y^{i+1} \geq y^i$.

[43] We will here consider the case where, at the initial equilibrium, no other type chooses the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$. Our argument can be easily generalized to the case where, at the initial equilibrium, more than two types are bunched together at the $(\widehat{y}, \widehat{c}, \widehat{e})$-bundle.

$\epsilon > 0$ and sufficiently small.[44] Clearly, type-i agents would not be tempted to choose the bundle $(y^{i+1}, c^{i+1}, e^{i+1})$ since their utility is strictly higher, by an amount equal to $(k^i - k^{i+1})\, g'(\widehat{e})\, \epsilon$, at the bundle $(y^i, \widehat{c}, \widehat{e})$ where their effort choice and well-being is the same as under the pre-reform equilibrium (where their bundle is $(\widehat{y}, \widehat{c}, \widehat{e})$). Type-$i+1$ agents would instead be just indifferent between the two bundles $(y^i, \widehat{c}, \widehat{e})$ and $(y^{i+1}, c^{i+1}, e^{i+1})$, which provide them the same utility that they enjoy under the pre-reform equilibrium (where their bundle is $(\widehat{y}, \widehat{c}, \widehat{e})$). Being indifferent between the two bundles $(y^i, \widehat{c}, \widehat{e})$ and $(y^{i+1}, c^{i+1}, e^{i+1})$, we can safely assume that all type-$i+1$ agents will choose the bundle $(y^{i+1}, c^{i+1}, e^{i+1})$. Notice however that, if at the initial equilibrium, type-$i+2$ agents were just indifferent between choosing the bundle $(y^{i+2}, c^{i+2}, e^{i+2})$, intended for them by the government, and the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$, after the reform they will be strictly better off (by an amount equal to $(k^{i+1} - k^{i+2})\, g'(\widehat{e})\, \epsilon$) choosing the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$. Thus, in order to preserve incentive-compatibility, the introduction of the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$ must be accompanied by an upward adjustment in c^{i+2} by $dc^{i+2} = (k^{i+1} - k^{i+2})\, g'(\widehat{e})\, \epsilon$. But once again, if at the initial allocation type-$i+3$ agents were just indifferent between choosing the bundle $(y^{i+3}, c^{i+3}, e^{i+3})$, intended for them by the government, and choosing the bundle $(y^{i+2}, c^{i+2}, e^{i+2})$, incentive-compatibility requires that c^{i+3} be adjusted upwards by the same amount as c^{i+2}. Replicating the same reasoning for all other types above $i+3$, one can conclude that offering the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$ requires to absorb a total amount of resources equal to $(k^{i+1} - k^{i+2})\, g'(\widehat{e}) \left(\sum_{j \geq i+2} \gamma^j\right) \epsilon$ in order to preserve incentive-compatibility. Thus, taking into account that total output in the economy would go up by $\gamma^{i+1}\theta^{i+1}\epsilon$ when type-$i+1$ agents switch from the initial bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ to the new bundle $(y^{i+1}, c^{i+1}, e^{i+1})$, the reform would allow the government to increase its net revenue by an amount equal to:

$$\left\{\gamma^{i+1}\theta^{i+1} - \left[\gamma^{i+1}k^{i+1} + \left(k^{i+1} - k^{i+2}\right)\left(\sum_{j \geq i+2} \gamma^j\right)\right] g'(\widehat{e})\right\} \epsilon. \tag{D 1}$$

Provided that the quantity defined in (D 1) is positive, i.e. when

$$\frac{\gamma^{i+1}\theta^{i+1}}{\left[\gamma^{i+1}k^{i+1} + \left(k^{i+1} - k^{i+2}\right)\left(\sum_{j \geq i+2} \gamma^j\right)\right] g'(\widehat{e})} > 1, \tag{D 2}$$

the pre-reform allocation, featuring bunching of type-i- and type-$i+1$ agents, is suboptimal. The reason is that, as we have shown above, one can implement a reform that, by separating the two types through an increase in the effort provided by type-$i+1$ agents, allows the government to obtain additional resources that can be used to raise social welfare.

Notice however that condition (D 2) is not a necessary condition to show that the initial

[44] Denote by $\overline{\theta}^{i,i+1}$ the average productivity of the agents that are bunched at the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$, i.e. $\overline{\theta}^{i,i+1} \equiv \left(\gamma^i\theta^i + \gamma^{i+1}\theta^{i+1}\right) / \left(\gamma^i + \gamma^{i+1}\right)$. Notice that it must necessarily be that $\widehat{y} = \overline{\theta}^{i,i+1}\widehat{e}$. Thus, y^{i+1} can also be equivalently re-expressed as $y^{i+1} = \widehat{y} + \epsilon\theta^{i+1} + \left(\theta^{i+1} - \overline{\theta}^{i,i+1}\right)\widehat{e}$ and y^i can be equivalently re-expressed as $y^i = \widehat{y} - \left(\overline{\theta}^{i,i+1} - \theta^i\right)\widehat{e}$.

allocation, featuring bunching of type-i- and type-$i+1$ agents, is suboptimal. In fact, another possibility to separate type-i- and type-$i+1$ agents is through a reform that achieves separation by lowering the effort exerted by type-i agents, without affecting their utility and without violating incentive-compatibility. For this purpose, consider the alternative reform where the government replaces the bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ with the bundle $(y^{i+1}, \widehat{c}, \widehat{e})$, where $y^{i+1} = \theta^{i+1}\widehat{e}$, and at the same time offers an additional bundle, which we will denote by (y^i, c^i, e^i), such that $e^i = \widehat{e} - \epsilon$, $c^i = \widehat{c} - \epsilon k^i g'(\widehat{e})$ and $y^i = (\widehat{e} - \epsilon)\theta^i$, with $\epsilon > 0$ and sufficiently small.[45] By construction, type-i agents would be just indifferent between the two bundles $(y^{i+1}, \widehat{c}, \widehat{e})$ and (y^i, c^i, e^i), which provide them the same utility that they enjoy under the pre-reform equilibrium (where their bundle is $(\widehat{y}, \widehat{c}, \widehat{e})$). Being indifferent between the two bundles $(y^{i+1}, \widehat{c}, \widehat{e})$ and (y^i, c^i, e^i), we can safely assume that all type-i agents will choose the latter bundle. Moreover, since type-$i+1$ agents are strictly better off at the initial bundle $(y^{i+1}, \widehat{c}, \widehat{e})$ than at the bundle (y^i, c^i, e^i), one can lower $\widehat{c}$ by $d\widehat{c} = (k^{i+1} - k^i)\, g'(\widehat{e})\,\epsilon < 0$ without worrying that type-$i+1$ agents are induced to switch to the bundle (y^i, c^i, e^i). But once $\widehat{c}$ is lowered by $d\widehat{c} = (k^{i+1} - k^i)\, g'(\widehat{e})\,\epsilon < 0$, one can afford to lower by the same amount, without violating incentive-compatibility, also the consumption available to agents choosing the bundles $(y^{i+2}, c^{i+2}, e^{i+2})$, $(y^{i+3}, c^{i+3}, e^{i+3})$, and so on. Thus, taking into account that total output in the economy would go down by $\gamma^i\theta^i\epsilon$ when type-i agents switch from the initial bundle $(\widehat{y}, \widehat{c}, \widehat{e})$ to the new bundle (y^i, c^i, e^i), the reform would allow the government to increase its net revenue by an amount equal to:

$$\left\{-\gamma^i\theta^i + \left[\gamma^i k^i + \left(k^i - k^{i+1}\right)\left(\sum_{j \geq i+1} \gamma^j\right)\right] g'(\widehat{e})\right\}\epsilon. \tag{D 3}$$

Provided that the quantity defined in (D 1) is positive, i.e. when

$$\frac{\gamma^i\theta^i}{\left[\gamma^i k^i + (k^i - k^{i+1})\left(\sum_{j \geq i+1} \gamma^j\right)\right] g'(\widehat{e})} < 1, \tag{D 4}$$

the pre-reform allocation, featuring bunching of type-i- and type-$i+1$ agents, is suboptimal. The reason is that one can implement a reform that, by separating the two types through an reduction in the effort provided by type-i agents, allows the government to obtain additional resources that can be used to raise social welfare.

Thus, putting together conditions (D 2) and (D 4), one can conclude that a necessary and sufficient condition for the suboptimality of the pre-reform allocation, featuring bunching of

[45]Notice that y^i can also be equivalently re-expressed as $y^i = \widehat{y} - \epsilon\theta^i - \left(\overline{\theta}^{i,i+1} - \theta^i\right)\widehat{e}$ and y^{i+1} can be equivalently re-expressed as $y^{i+1} = \widehat{y} + \left(\theta^{i+1} - \overline{\theta}^{i,i+1}\right)\widehat{e}$.

type-i- and type-$i+1$ agents, is

$$\frac{\gamma^{i+1}\theta^{i+1}}{\left[\gamma^{i+1}k^{i+1}+\left(k^{i+1}-k^{i+2}\right)\left(\sum_{j\geq i+2}\gamma^{j}\right)\right]g'\left(\widehat{e}\right)}>\frac{\gamma^{i}\theta^{i}}{\left[\gamma^{i}k^{i}+\left(k^{i}-k^{i+1}\right)\left(\sum_{j\geq i+1}\gamma^{j}\right)\right]g'\left(\widehat{e}\right)}, \tag{D 5}$$

or, equivalently, condition (22) in the main document.