

Household Specialization and Competition for Promotion*

Spencer Bastani[†] Lisa Dickmanns[‡] Thomas Giebe[§] Oliver Gürtler[¶]

May 2, 2022

Abstract

We analyze equally competitive spouses competing for promotion in separate workplaces and show that an asymmetric equilibrium featuring household specialization can emerge. Examples where the asymmetric equilibrium is welfare-superior to the symmetric equilibrium are highlighted. By investing heavily in the career of only one spouse, families reduce the intensity of the rat race of the working environment and obtain less risky consumption opportunities. Our findings suggest that specialization can reflect an efficient response to the competitiveness of the labor market and may arise even when all workers have equal opportunities to succeed in the labor market.

Keywords: contest theory, gender equality, family, household, competition

JEL classification: C72, D13, J16, J71, M51, M52

*All authors gratefully acknowledge financial support from the Jan Wallander and Tom Hedelius Foundation (grant no. P18-0208). Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy EXC 2126/1-390838866.

[†]Institute for Evaluation of Labour Market and Education Policy (IFAU) and Research Institute of Industrial Economics (IFN); Uppsala Center for Fiscal Studies (UCFS); Uppsala Center for Labor Studies (UCLS); CESifo. E-mail: spencer.bastani@ifau.uu.se.

[‡]Department of Economics, University of Cologne, Germany. E-mail: l.dickmanns@gmail.com

[§]Department of Economics and Statistics, School of Business and Economics, Linnaeus University, Sweden. E-mail: thomas.giebe@lnu.se.

[¶]Department of Economics, University of Cologne, Germany. E-mail: oliver.guertler@uni-koeln.de

1 Introduction

A large literature in labor economics documents gender gaps in labor market outcomes, emphasizing differences in wages, working hours, employment rates, and occupations. Despite a strong convergence process over the last half-century, substantial gender differences in the labor market remain (Olivetti and Petrongolo 2016). There is an intense discussion among academics and policymakers alike regarding the “last chapter” (Goldin 2014) of gender convergence. In this paper, we wish to contribute to this discussion by highlighting how gender gaps can arise in response to the competitiveness of the labor market even when men and women are equally competitive and face equal opportunities to succeed in the labor market. Our focus is on settings where career investments are coordinated at the household level and firms use promotions as an incentive mechanism.

We set up a theoretical model with two identical two-earner families consisting of two identical spouses. Each spouse in the first family competes for promotion against a spouse in the second family. In this stylized but tractable model, we first show that asymmetric equilibria featuring household specialization generally can emerge. We then illustrate that the specialization equilibrium can deliver higher welfare to both households as compared to when spouses in both families adopt the same competitive effort.

The intuition behind the natural emergence of household specialization is twofold. First, the asymmetric equilibrium reduces the intensity of the rat race of the labor market (as the promotion competition within each firm becomes less intense) implying that both households save on effort costs. Second, a situation where only one spouse exerts high effort provides smoothing of family consumption since intermediate events (where one spouse in each household gets promoted) become more likely.¹

In the traditional labor supply model, household specialization typically arises as long as one partner has a comparative advantage in market work (see Pollak 2013 for a discussion). We show that household specialization can arise in a setting with ex-ante *identical* spouses participating in promotion tournaments which is a prevalent incentive structure in competitive labor markets. In this way, our paper relates to Francois (1998) who also considers ex-ante identical men and women but focuses on explaining gender discrimination as an equilibrium outcome in a setting where men and women select into different jobs (in our setting, in contrast, men and women work in identical jobs).²

¹The consumption insurance channel has previously been highlighted in a non-tournament setting by, e.g., Blundell et al. (2018).

²Our paper can be viewed as adding a new angle to the literature on labor market investments within families. One prominent strand of this literature discusses the “family investment hypothesis” and how credit constraints (in the case of immigrant workers) can imply labor market behavior where a “primary worker” engages in investment activities and the other partner undertakes activities that finance consumption (see, e.g., Baker and Benjamin 1997 and Cobb-Clark and Crossley 2004).

2 The model

Following Lazear and Rosen (1981), each worker exerts effort to produce output and the worker with the highest output in the tournament gets promoted and wins a prize w_P , whereas the non-promoted worker receives w_{NP} . Output of each worker is equal to $y = e + \epsilon$, where e is individual effort and ϵ is a random component. The ϵ are assumed to be independently and identically distributed with PDF f and CDF F .

The distinguishing feature of our setup is that we embed the Lazear-Rosen tournament in a household environment. More specifically, we consider two families 1 and 2 and two identical firms A and B . In each family, one member works in firm A , while the other member works in firm B . We denote by ik the spouse in family $i \in \{1, 2\}$ who works in firm $k \in \{A, B\}$, and by il the spouse in family i who works in firm $l \in \{A, B\}$, where $k \neq l$. Similarly, we denote by jk the spouse in family $j \in \{1, 2\}$, $j \neq i$, who works in firm $k \in \{A, B\}$ and by jl the spouse in family j who works in firm $l \in \{A, B\}$, $k \neq l$. The tournament prize structure is the same in both firms. Total family income is equal to the sum of the prizes, which, given the two possible prize levels, admits four different configurations of family income given by the pairs (w_P, w_P) , (w_P, w_{NP}) , (w_{NP}, w_P) , and (w_{NP}, w_{NP}) .

Adopting the unitary model of household decision-making (see, e.g., Boskin and Sheshinski 1983 and Kleven et al. 2009), household i 's utility is

$$U(b_i, e_{ik}, e_{il}) = u(b_i) - c(e_{ik}) - c(e_{il}), \quad (1)$$

where b_i denotes family i 's total consumption, and $e_{ik} \geq 0$ and $e_{il} \geq 0$ denote the efforts exerted by the spouses in family i .³ Moreover, u is non-decreasing and strictly concave and c is non-decreasing and strictly convex, satisfying $c(0) = 0$ and $c'(0) = 0$.

The assumption that utility is nonlinear in consumption and depends on total household disposable income is key to our analysis as it implies a decreasing return for a family to have both spouses being highly successful in the labor market.

Let $\Delta e_k = e_{ik} - e_{jk}$ be the effort difference in firm k and $\Delta e_l = e_{il} - e_{jl}$ the effort difference in firm l . Moreover, let $\Delta u_P = u(2w_P) - u(w_P + w_{NP})$ be the gain in the utility from consumption of going from one to two promoted family members, and $\Delta u_{NP} = u(w_P + w_{NP}) - u(2w_{NP})$ be the gain of going from zero to one promoted family member. We also define $\Delta u = \Delta u_P - \Delta u_{NP} = u(2w_P) + u(2w_{NP}) - 2u(w_P + w_{NP})$, which is negative, by virtue of the strict concavity of u . In other words, the first promotion within the family is more valuable than the second.

³We recognize that there are other models of family-decision making (see the discussion in Chiappori and Luebel 2015). The unitary model gives us tractability and serves as an important benchmark in the family economics literature.

The expected utility of family i can be written as:

$$G(\Delta e_k)G(\Delta e_l)[\Delta u_P - \Delta u_{NP}] + [G(\Delta e_k) + G(\Delta e_l)]\Delta u_{NP} + u(2w_{NP}) - c(e_{ik}) - c(e_{il}). \quad (2)$$

Notice that G is the CDF of $\epsilon_{js} - \epsilon_{is}$, $s \in \{k, l\}$, and $G(\Delta e_k)$ is the probability that family i wins the firm- k tournament and $G(\Delta e_l)$ is the probability that family i wins the firm- l tournament. The baseline utility from consumption is $u(2w_{NP})$ and the second term above gives the increase in utility from winning either of the two tournaments, whereas the first term gives the extra increase in utility when winning both tournaments. Household i jointly chooses e_{ik} and e_{il} in order to maximize (2). Family j faces a problem with the same structure in which the probabilities $G(\Delta e_k)$ and $G(\Delta e_l)$ are replaced by $G(-\Delta e_k)$ and $G(-\Delta e_l)$, respectively, and the maximization is carried out with respect to e_{jk} and e_{jl} instead.

The outcome of the firm k tournament is determined by e_{ik} and e_{jk} abiding the first-order conditions

$$g(\Delta e_k) \left[G(\Delta e_l)\Delta u_P + (1 - G(\Delta e_l))\Delta u_{NP} \right] = c'(e_{ik}) \quad (3)$$

$$g(-\Delta e_k) \left[G(-\Delta e_l)\Delta u_P + (1 - G(-\Delta e_l))\Delta u_{NP} \right] = c'(e_{jk}). \quad (4)$$

2.1 The possibility of asymmetric equilibria

The possibility of asymmetric equilibria is not immediately apparent in our fully symmetric model. However, as we will see, asymmetric household specialization equilibria can arise. We focus on such equilibria where the *total* effort in both families is the same:

$$e_{ik} + e_{il} = e_{jk} + e_{jl} \iff e_{ik} - e_{jk} = -(e_{il} - e_{jl}) \iff \Delta e_k = -\Delta e_l \neq 0. \quad (5)$$

Conditional on the efforts of family j , if spouse k in household i specializes in market work (in the sense of exerting a high effort in his/her promotion tournament), it must be the case that spouse l in household i specializes in household work or leisure (in the sense of exerting a low effort in his/her promotion tournament).

Assuming f is uni-modal and symmetric around zero, which implies that g also is uni-modal and symmetric around zero, and $c(e) = de^2$ with $d > 0$, equations (3) and (4) can be re-written, exploiting (5), as

$$g(\Delta e_k) \left[(1 - G(\Delta e_k))\Delta u_P + G(\Delta e_k)\Delta u_{NP} \right] = 2de_{ik} \quad (6)$$

$$g(\Delta e_k) \left[G(\Delta e_k)\Delta u_P + (1 - G(\Delta e_k))\Delta u_{NP} \right] = 2de_{jk}. \quad (7)$$

Subtracting (7) from (6) and re-arranging yields:

$$g(\Delta e_k) \left[G(\Delta e_k) - \frac{1}{2} \right] = \frac{d}{-\Delta u} \Delta e_k, \quad (8)$$

where we recall that $\Delta u = \Delta u_P - \Delta u_{NP} < 0$ so the RHS is nonnegative for $\Delta e_k \geq 0$. We have the following result:

Proposition 1. *If g is continuous on \mathbb{R}^+ and $g(0) > \sqrt{\frac{d}{-\Delta u}}$, then there is $\Delta e > 0$ such that (8) holds.*

Proof. Based on (8), we define $H(\Delta e) = g(\Delta e) (G(\Delta e) - 1/2) - \alpha \cdot \Delta e$ where $\alpha = \frac{d}{-\Delta u} > 0$. Notice that $H(\Delta e) < 0$ when $\Delta e \rightarrow \infty$ (as $G(\Delta e)$ and $g(\Delta e)$ are bounded). What remains to show is that $H(\Delta e) > 0$ for some $\Delta e > 0$. We consider the point $\Delta e = \varepsilon$ where $\varepsilon > 0$ is small such that $g(\varepsilon) > \sqrt{\alpha}$. We get:

$$\begin{aligned} H(\varepsilon) &= g(\varepsilon) \left(\int_{-\infty}^0 g(t) dt + \int_0^\varepsilon g(t) dt - 1/2 \right) - \alpha \varepsilon = g(\varepsilon) \left(\int_0^\varepsilon g(t) dt \right) - \alpha \varepsilon \\ &= g(\varepsilon) g(\xi) \varepsilon - \alpha \varepsilon > 0, \end{aligned}$$

for some $\xi \in (0, \varepsilon)$. The last inequality follows because $g(0) > \sqrt{\alpha}$ and g continuous implies $g(x) > \sqrt{\alpha}$ for all $x \in [0, \varepsilon]$. \square

2.2 The symmetric equilibrium

A symmetric equilibrium \hat{e} satisfies $\Delta e_k = \Delta e_l = 0$. Insertion into either (3) or (4) yields

$$g(0) \frac{1}{2} [\Delta u_P + \Delta u_{NP}] = c'(\hat{e}). \quad (9)$$

As this equation readily has a solution, a symmetric equilibrium candidate generally exists. Notice that \hat{e} is the effort level chosen by both spouses in both families.

2.3 Welfare comparison between the two equilibria

Whether the asymmetric or symmetric equilibrium delivers higher utility to families depends on how large the *expected utility from consumption* and *total effort costs* are in each equilibrium, recalling the definition of household welfare in (1). The expected consumption utility for family i in a *symmetric equilibrium* is:

$$\frac{1}{4} [u(2w_P) + u(2w_{NP})] + \frac{1}{2} u(w_P + w_{NP}). \quad (10)$$

In contrast, when playing the *asymmetric equilibrium*, it is:

$$G(\Delta e_k)(1 - G(\Delta e_k)) \cdot [u(2w_P) + u(2w_{NP})] + [(G(\Delta e_k))^2 + (1 - G(\Delta e_k))^2] \cdot u(w_P + w_{NP}). \quad (11)$$

We have $G(\Delta e_k)(1 - G(\Delta e_k)) \leq \frac{1}{4}$ and since probabilities add up to one, it follows that $[(G(\Delta e_k))^2 + (1 - G(\Delta e_k))^2] \geq \frac{3}{4} > \frac{1}{2}$. By the strict concavity of u , expression (11) will thus be strictly larger than (10). Thus, the asymmetric equilibrium always provides “smoothing” of total household consumption as it attaches higher probabilities to outcomes featuring one promoted spouse per household.

Regarding total effort costs, in Appendix A.3, we show for the Uniform distribution that if d is sufficiently large, and u is not too concave, then both efforts in the asymmetric equilibrium are smaller than the symmetric equilibrium effort (provided they co-exist), implying that welfare is higher through both the consumption and effort channel.⁴

2.4 Numerical example

We now provide a numerical example illustrating that the asymmetric equilibrium can welfare-dominate the symmetric equilibrium when they both co-exist (see the Appendix for details). Assume the noise terms are uniformly distributed on $[-1/2, 1/2]$. Without loss of generality, we assume family i puts in more effort into the firm- k tournament than family j , namely, $\Delta e_k > 0$.

Table 1 presents the results for $c(e) = e^2$. We fix $u(2w_{NP}) = 2$, $u(w_{NP} + w_P) = 4$ and consider variation in $u(2w_P)$, letting it take on the three values 4.1, 4.5 and 5. Note that for $u(2w_P) = 5$ (third line), equation (8) has the unique solution $\Delta e_k = 0$. In the other two examples, (8) has two solution candidates, i.e., $\Delta e_k = 0$ and $\Delta e_k > 0$, although only one of the candidates is an equilibrium for $u(2w_P) = 4.1$. In the example with $u(2w_P) = 4.5$, we can compare the two equilibria as they exist simultaneously, and we see that the asymmetric equilibrium has lower total effort cost *and* higher expected utility from consumption, showing that the asymmetric equilibrium welfare-dominates the symmetric equilibrium.

⁴Generally, given asymmetric efforts of household j , household i faces asymmetric tournaments in both firms, which typically have lower effort than symmetric tournaments. On the other hand, asymmetric efforts imply higher total effort cost due to convex cost functions.

Table 1: Numerical example

$u(2w_P)$	symmetric equilibrium			asymmetric equilibrium		
	e_{sym}	total cost	$E[u(b)]$	(e_H, e_L)	total cost	$E[u(b)]$
4.1	–	–	–	(0.516, 0.157)	0.291	3.690
4.5	0.625	0.781	3.625	(0.595, 0.353)	0.478	3.693
5	0.750	1.125	3.750	–	–	–

Note: Illustration of the possibility to either have only an asymmetric equilibrium (first row), only a symmetric equilibrium (third row), or both existing at the same time (second row). Total cost is equal to $2c(e_{sym})$ in the case of a symmetric equilibrium, and equal to $c(e_H) + c(e_L)$ in the case of an asymmetric equilibrium. Second-order conditions have been verified.

3 Concluding remarks

Promotion tournaments are prevalent incentive structures in competitive labor markets. In a fully symmetric model with ex-ante identical spouses, we have shown that household specialization can arise when spouses engage in such tournaments. The specialization equilibrium can welfare-dominate the symmetric equilibrium as it allows both households to save on effort costs and provides consumption-smoothing benefits.

Our setting is stylized, yet delivers the important result that household specialization can arise when spouses are equally competitive and face the same labor market circumstances. It shows that household specialization can reflect an efficient response to firms' incentive mechanisms. Future work could explore if the mechanisms highlighted by our model would remain relevant in more general settings.

In our paper, the gender identity of each spouse is unspecified. However, given gender patterns of household specialization (e.g., motherhood tends to favor men investing more in their careers), our analysis suggests that substantial gender differences in labor market outcomes may remain even when men and women face equal opportunities to succeed in the labor market. As our results originate from spousal differences in effort, where "high effort" can be interpreted as working "long" or "continuous" hours, the mechanisms that we highlight are consistent with the challenges of the last chapter of gender convergence (Goldin 2014).

References

- Baker, M. and Benjamin, D. (1997). The role of the family in immigrants' labor-market activity: An evaluation of alternative explanations. *American Economic Review*, pages 705–727.
- Blundell, R., Pistaferri, L., and Saporta-Eksten, I. (2018). Children, time allocation, and consumption insurance. *Journal of Political Economy*, 126(S1):S73–S115.
- Boskin, M. J. and Sheshinski, E. (1983). Optimal tax treatment of the family: Married couples. *Journal of Public Economics*, 20(3):281–297.
- Chiappori, P. and Lewbel, A. (2015). Gary Becker's a theory of the allocation of time. *Economic Journal*, 125(583):410–442.
- Cobb-Clark, D. and Crossley, T. F. (2004). Revisiting the family investment hypothesis. *Labour Economics*, 11(3):373–393.
- Francois, P. (1998). Gender discrimination without gender difference: Theory and policy responses. *Journal of Public Economics*, 68(1):1–32.
- Goldin, C. (2014). A grand gender convergence: Its last chapter. *American Economic Review*, 104(4):1091–1119.
- Kleven, H. J., Kreiner, C. T., and Saez, E. (2009). The optimal income taxation of couples. *Econometrica*, 77(2):537–560.
- Lazear, E. P. and Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy*, 89(5):841–864.
- Olivetti, C. and Petrongolo, B. (2016). The evolution of gender gaps in industrialized countries. *Annual Review of Economics*, 8(1):405–434.
- Pollak, R. A. (2013). Allocating household time: When does efficiency imply specialization? *National Bureau of Economic Research*, (19178).

A Additional results and derivations

A.1 Details on the triangular distribution

In our numerical example, we assume that f is the PDF of the Uniform distribution on $[-1/2, 1/2]$. This implies that the differences of noise terms follow a Triangular distribution with PDF g and CDF G given by

$$g(\Delta e) = \begin{cases} \Delta e + 1, & \text{for } -1 \leq \Delta e < 0 \\ 1, & \text{for } \Delta e = 0 \\ 1 - \Delta e, & \text{for } 0 < \Delta e \leq 1 \\ 0, & \text{for } \Delta e \notin [-1, 1] \end{cases}$$

$$G(\Delta e) = \begin{cases} 0, & \text{for } \Delta e < -1 \\ \frac{(\Delta e + 1)^2}{2}, & \text{for } -1 \leq \Delta e < 0 \\ \frac{1}{2}, & \text{for } \Delta e = 0 \\ 1 - \frac{(1 - \Delta e)^2}{2}, & \text{for } 0 < \Delta e < 1 \\ 1, & \text{for } \Delta e \geq 1. \end{cases}$$

A.2 Deriving efforts in the numerical example

Given the assumptions in the beginning of Section 2.4, the symmetric equilibrium effort \hat{e} is given by the solution of (9):

$$\hat{e} = \frac{1}{4d}(\Delta u_P + \Delta u_{NP}). \quad (\text{A.1})$$

Supposing that the asymmetric candidate satisfies $\Delta e_k \in [0, 1]$ we can re-write (8) as:

$$(1 - \Delta e_k) \left[\frac{(1 - \Delta e_k)^2}{2} - \frac{1}{2} \right] = \frac{d}{\Delta u} \Delta e_k$$

which has the obvious solution $\Delta e_k = 0$ (the symmetric equilibrium candidate). The unique solution satisfying $\Delta e_k \in (0, 1]$ is:

$$\Delta e_k^{\text{asym}} = \frac{3}{2} - \sqrt{\frac{1}{4} - \frac{2d}{\Delta u}}, \quad (\text{A.2})$$

provided $-\Delta u > d$. Denote the high effort in the asymmetric equilibrium by e_H and the low effort by e_L , such that $\Delta e_k^{\text{asym}} = e_H - e_L > 0$. Plugging Δe_k^{asym} into (6) allows us to solve for $e_H = e_{ik}^{\text{asym}}$ (which, by construction, is identical to e_{jl}^{asym}):

$$e_H = e_{ik}^{\text{asym}} = \frac{\Delta u}{4d} (1 - \Delta e_k^{\text{asym}})^3 + \frac{\Delta u_{NP}}{2d} (1 - \Delta e_k^{\text{asym}}). \quad (\text{A.3})$$

Here we notice that the first term is negative and the second term is positive given that $(1 -$

$\Delta e_k^{\text{asym}} \in [0, 1]$ and $\Delta u < 0$. The low effort e_L is found by solving $\Delta e_k^{\text{asym}} = e_H - e_L$.

A.3 Welfare comparison in the case of a Uniform distribution

Here we provide some additional results for the case in which F follows a Uniform distribution.

Proposition 2. *Impose the distributional assumptions of Section 2.4. Then, if d is sufficiently large, and u is not too concave:*

(i) $e_{ik}^{\text{asym}} = e_{jl}^{\text{asym}} < \hat{e}$ and $e_{jk}^{\text{asym}} = e_{il}^{\text{asym}} < \hat{e}$.

(ii) *The asymmetric equilibrium provides higher welfare to both families as compared to the asymmetric equilibrium in the cases where both equilibria exist.*

Proof. We begin with Part (i). Notice that when d approaches its upper bound, $-\Delta u$, we have that $\Delta e_{ik}^{\text{asym}} \rightarrow 0$ (see equation A.2) and $e_{ik}^{\text{asym}} \rightarrow \hat{e}$ (recall (A.1)). Now consider a value of d slightly lower than $-\Delta u$, namely, $d = -\Delta u - \delta$ where $\delta > 0$ is small. This implies that $\Delta e_{ik}^{\text{asym}} = \varepsilon$ where $\varepsilon > 0$ also is small and is a function of δ . We then have from (A.3) that:

$$e_{ik}^{\text{asym}} = \frac{\Delta u}{4d} (1 - \varepsilon)^3 + \frac{\Delta u_{NP}}{2d} (1 - \varepsilon) \approx \frac{\Delta u}{4d} (1 - 3\varepsilon) + \frac{\Delta u_{NP}}{2d} (1 - \varepsilon), \quad (\text{A.4})$$

where the approximation follows by neglecting terms of order ε^2 and ε^3 . Subtracting (A.1) from the approximated effort in (A.4) and re-arranging yields:

$$e_{ik}^{\text{asym}} - \hat{e} \approx -\frac{\varepsilon}{4d} (3\Delta u_P - \Delta u_{NP}) \quad (\text{A.5})$$

which is negative provided $3\Delta u_P > \Delta u_{NP}$. This condition amounts to requiring u not to be too concave. To see this, notice that:

$$\begin{aligned} 3\Delta u_P - \Delta u_{NP} &= 3 \left[u(2w_P) - u(w_P + w_{NP}) \right] - \left[u(w_P + w_{NP}) - u(2w_{NP}) \right] \\ &= 3u(2w_P) + u(2w_{NP}) - 4u(w_P + w_{NP}), \end{aligned}$$

which will be strictly positive for any u that is not too concave. Finally, notice that by virtue of our assumption (without loss of generality) that family i puts in more effort into the firm- k tournament than family j , we also have that $e_{jk}^{\text{asym}} = e_{il}^{\text{asym}} < \hat{e}$. Thus, in the asymmetric equilibrium, both spouses in both families save on effort costs compared to the symmetric equilibrium.

Regarding Part (ii), this follows from Part (i) combined with the fact that the expected utility from consumption is strictly higher in the asymmetric equilibrium (see Section 2.3) due to the strict concavity of u . □