# OVERCONFIDENCE AND GENDER EQUALITY IN THE LABOR MARKET*

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**Abstract**

Gender differences in overconfidence have been extensively documented in the empirical literature, but the implications for labor market outcomes are not well understood. In this paper, we analyze how men's relatively higher overconfidence, combined with competitive job incentives, affects gender equality in the labor market and discuss policy implications. The vehicle of analysis is a promotion-signaling model in which wages are realistically determined by market forces. We find that overconfident workers exert more effort to be promoted, and even though they have lower expected ability conditional on promotion, they are more likely to be promoted and experience superior wage growth. Because overconfident workers compete fiercely, they incur higher effort costs and discourage their peers, and we find that overconfidence can be either self-serving or self-defeating.

**Keywords:** overconfidence, promotion, competition, gender gap, tournament, theory
**JEL** classification: C72, D91, J16, J24, M51, M52

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# 1 Introduction

Labor market outcomes for men and women have converged remarkably in recent decades. Cultural norms have changed, workplaces have become more family-friendly, and governments have offered more generous childcare and parental leave policies that allow workers to combine work and family life. Nevertheless, significant gender gaps remain in the labor market. This is particularly the case in high-skilled jobs and at the top of the earnings distribution. At the beginning of their careers, highly skilled men and women tend to have similar earnings, but over time significant gender gaps emerge (Noonan et al., 2005, Manning and Swaffield, 2008, Bertrand et al., 2010, Azmat and Ferrer, 2017). As important explanations for differences in career advancement, labor economists point to long and "particular" work hours, psychological traits that influence competitive behavior, and child-rearing obligations (Goldin, 2014).

A growing body of literature suggests that men and women behave differently in the face of competition. For example, women have been found to be less willing to self-select into competitive situations and to accept pay systems that base income on performance relative to peers (Gneezy et al., 2003, Niederle and Vesterlund, 2007, Croson and Gneezy, 2009). Women are also less likely to ask for a promotion (Bosquet et al., 2019), tend to ask for lower salaries (Säve-Söderbergh, 2019), and are more likely to be asked and more likely to volunteer for tasks that count little toward promotion (Babcock et al., 2017). Men have been found to be more likely to sabotage colleagues and to compete harder with women (Dato and Nieken, 2014). In addition, certain job descriptions and tasks have been found to attract men and deter women when associated with male stereotypes (Dreber et al., 2014, Flory et al., 2015, 2021).

In this paper, we examine the relatively higher level of overconfidence among men and the mechanisms through which it affects promotion outcomes and gender equality in the labor market. Gender-differences in overconfidence have been empirically identified in a number of different contexts (Camerer and Lovallo, 1999, Niederle and Vesterlund, 2007, Hoffman and Burks, 2020).[1] In a famous paper, Niederle and Vesterlund (2007) found that men were almost twice as likely to select into a competitive pay scheme, a result they attribute to gender differences in overconfidence and preferences for competitive situations. More recently, van Veldhuizen (2022) has found that overconfidence and risk aversion, rather than competitive preferences, explain self-selection into competitive pay. According to Adamecz-Völgyi and Shure (2022), overconfidence among men could explain as much as 5-11 percent of the gender employment gap in top jobs.[2]

To study the labor market effects of overconfidence and its implications for gender equality, we construct a promotion-signaling model in which work effort, broadly interpreted as effec-

[1] Sarsons and Xu (2021) argue that male overconfidence can alternatively be interpreted as women having better judgment for the limits of their expertise.

[2] Overconfidence has also been shown to be evolutionarily stable as overestimating one's ability can serve motivational and ego-utility reasons (Waldman, 1994, Zimmermann, 2020). Since effort and ability are usually complements, a higher confidence in one's ability motivates higher effort (Bénabou and Tirole, 2002), which is supported by empirical evidence (Chen and Schildberg-Hörisch, 2019, Bruhin et al., 2022).

tive hours worked, affects labor market outcomes. The reason we focus on competition for promotion is that promotions are important drivers of individual wage growth (see, e.g., Baker et al., 1994b) and promotion tournaments serve as important incentive systems in most firms and organizations.[3] The reason we focus on the promotion-signaling model is that it realistically captures the fact that incumbent employers typically have superior information about their employees and that outside firms base their hiring attempts on observable signals (Waldman, 1990, Acemoglu and Pischke, 1998). Internal promotions serve as a signal of worker ability in the labor market. Employees' observable career progression forms the basis of external wage offers, which shape the wage-setting policy of the incumbent employer.[4] Empirical support for the signaling role of promotions in wage determination is provided by, for example, DeVaro and Waldman (2012), Bognanno and Melero (2016), and Cassidy et al. (2016).[5]

The contribution of our paper is threefold. First, we embed overconfidence in one's own abilities in a promotion-signaling model and show how overconfidence shapes human capital investment in the early career. Second, we link our theoretical results to existing empirical evidence to examine how male overconfidence can explain gender differences in working hours, human capital accumulation, wages, the representation of women in high-level jobs, and the educational attainment of promoted managers. Third, we discuss policy implications and how working time regulation and employment protection can influence the effects of overconfidence on labor market outcomes.

The details of our model are as follows. We consider two workers in the early stages of their careers who make effort choices that have consequences for human capital formation and the likelihood of being promoted. The effort choices can be interpreted as effective working time that leads to human capital accumulation through learning-by-doing.[6] Each worker has an inherent ability drawn from a statistical distribution common to both workers. The key assumption of our setup is that one of the workers is *overconfident* in the sense that the overconfident worker (hereafter referred to as "him") perceives his ability to be drawn from a distribution that is superior to his actual distribution. In all other respects, the workers are identical. In particular, they have the same preferences and the same ex ante chances of being promoted.

There are two job levels in every firm. The first job level is an entry-level job, while the second is a high-level job, such as a managerial position, which is filled by promoting one of

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[3]The prominent role of promotion tournaments as incentive systems in firms has been established both theoretically and empirically (Lazear and Rosen, 1981, Green and Stokey, 1983, Malcomson, 1984, Baker et al., 1994a,b, Prendergast, 1999, Bognanno, 2001, DeVaro, 2006, DeVaro et al., 2019).

[4]Starting with Waldman (1984), asymmetric learning in the labor market combined with promotion signaling has been studied theoretically and empirically as an important driver of labor market outcomes. Promotion signaling models are analyzed in Bernhardt (1995), Zábojník and Bernhardt (2001), Ghosh and Waldman (2010), DeVaro and Waldman (2012), Zábojník (2012), Waldman (2013), Gürtler and Gürtler (2015), Waldman (2016), DeVaro et al. (2018), Gürtler and Gürtler (2019).

[5]One way to think intuitively about promotion signaling in modern labor markets is that promotions are visible on social media and hiring platforms, potentially triggering hiring responses.

[6]For empirical evidence on learning and learning-by-doing see, e.g., De Grip et al. (2016), Stinebrickner et al. (2019), Caplin et al. (2022), James et al. (2022).

the entry-level workers after he or she has gained experience in the firm. Worker productivity is more important for the firm in the high-level job. The incumbent firm observes the worker's performance in the entry-level job and forms beliefs about the worker's unobservable ability and effort level. These beliefs form the firm's expectations about the productivity that each worker would have in the managerial position.

Our main results can be summarized as follows. First, given the production technology and competition for workers from outside firms, we show that the promotion rule is unbiased in the sense that it is in the best interest of the incumbent firm to promote the worker who has the highest expected productivity in the managerial position. This is consistent with recent empirical evidence. For example, Azmat and Ferrer (2017) report that "once performance is accounted for, the gender gap in partnership status is no longer statistically significant", suggesting that promotion decisions are primarily driven by actual performance, and Bender et al. (2018) find that the human capital of managers (promoted workers) plays a large role in firm productivity.

Our second result concerns the effect of overconfidence on career investment. In principle, overconfidence could lead to either lower or higher promotion efforts. On the one hand, a worker who perceives himself as more productive may not feel the need to work as hard, since his perceived probability of being promoted is higher at any level of effort. On the other hand, if effort and ability are complements in the production function (Bénabou and Tirole, 2002), higher perceived ability implies that the overconfident worker overestimates the marginal impact of effort on the probability of promotion, justifying higher effort. We show that, under our assumptions, the latter effect dominates, implying that the overconfident worker exerts more effort (in line with empirical evidence, e.g., Chen and Schildberg-Hörisch, 2019, Bruhin et al., 2022).

The higher effort of the overconfident worker has several implications for labor market outcomes. Not only does the higher effort imply that the overconfident worker is more likely to be promoted, but interestingly, it also implies that he earns a higher wage later in his career, conditional on the job level. Interestingly, the overconfident worker has a lower expected ability conditional on promotion. However, because of his higher effort, he acquires more transferable human capital through learning-by-doing and is therefore more productive. The overconfident worker may either be better off, consistent with the idea of overconfidence as a "self-serving bias" (see, e.g., Bénabou and Tirole, 2002, Zimmermann, 2020), or worse off, due to the costs of excessive effort.

Our findings are consistent with a number of gender differences highlighted in the empirical literature:

- Men's and women's earnings are equal at the beginning of their careers, but then diverge due to men's longer working hours and faster accumulation of work experience (Landers et al., 1996, Azmat and Ferrer, 2017, Goldin, 2014).

- Women have lower promotion rates than men and are underrepresented at higher levels of the corporate hierarchy (Goldin, 2014, Azmat and Ferrer, 2017, Cook et al., 2021).

- Conditional on job level, women have lower wages than men (Blau and Kahn, 2017).

- Women at higher levels of the corporate hierarchy tend to be better educated than their male counterparts (Heyman et al., 2020, Keloharju et al., 2022, Campbell and Hahl, 2022).[7]

Overconfidence is neither the only nor necessarily the most important explanation for gender gaps in the labor market. Nevertheless, it is a psychological trait that has received considerable attention and support in the recent empirical literature. Our approach is to take overconfidence as given and to examine its implications for labor market outcomes when firms use competitive promotion incentives. In particular, our results are obtained under the assumption that overconfidence is the only difference between workers. In particular, workers are equally productive (in the sense of having identical ability distributions) and have identical effort costs and competitive preferences. Thus, we identify mechanisms that are relevant even if all workers have equal opportunities to succeed in the labor market and women do not "shy away" from competition.

We conclude the paper by discussing some policy implications of our results. We show that any policy that imposes an appropriately chosen upper bound on workplace effort would completely eliminate any gender differences driven by overconfidence. We also discuss the potential effects of firm-level "confidence management" policies that could affect the confidence levels of workers in the firm.

In terms of the literature to date, only a handful of papers have examined the role of overconfidence in promotion competition from a theoretical perspective. Deng et al. (2020) consider an employee's confidence in another employee's ability as well as the firm's confidence management and information disclosure policies. Santos-Pinto (2010, 2021, 2022) and Santos-Pinto and Sekeris (2022) theoretically study the effects of overconfidence in tournaments. In these papers, prizes (wages) are either exogenous, as in many tournament models, or they are endogenous but do not depend on who wins the tournament. In contrast, our model of promotion signaling allows wage offers to depend on the identity of the worker. This is a key feature as it allows us to explain gender wage differences conditional on job level, which is a recurring finding in the empirical literature.

The plan of the paper is as follows. Section 2 presents the model, and Section 3 characterizes the equilibrium and provides numerical examples. Section 4 discusses policy implications,

[7] We thank Joacim Tåg for pointing out that their working paper version Keloharju et al. (2016) shows in Table 2 that Swedish female CEOs have a higher share of university education than their male counterparts. Relatedly, Heyman et al. (2020) show theoretically that a 'gender-specific career hurdle' implies that female managers will have higher ability than male managers, as only women from the high end of the talent distribution will invest in management careers.

and Section 5 concludes. The Online Supplementary Material contains all derivations and proofs.

## 2 Model

We consider a competitive labor market with $n \geq 3$ identical firms. There are two periods, $t \in \{1, 2\}$, representing the early and late stages of workers' careers. In period 1, one of the firms (hereafter the *incumbent*) hires two workers, $A$ and $B$. Each worker $i \in \{A, B\}$ produces output through a combination of ability $\Theta_i$ and effort $e_i$. Following, e.g., Holmström (1982), we assume symmetric uncertainty about ability, i.e., ability $\Theta_i$ is a random variable and its realization, denoted by $\theta_i$, is not observable by any firm or worker (not even worker $i$). The ability of each worker is uniformly distributed on $[0, 1]$ with cdf $F$ and pdf $f$.

Worker $A$ is assumed to be *overconfident*. Specifically, worker $A$ believes that his ability follows a distribution with cdf $\hat{F}(x) = x^{\gamma}$, $\gamma > 1$, on support $[0, 1]$ and corresponding pdf $\hat{f}$. This probability distribution, which we call the *subjective* ability distribution of $A$, first-order stochastically dominates the actual ability distribution, giving greater weight to higher ability.[8] Note that larger values of the overconfidence parameter $\gamma$ correspond to a greater degree of overconfidence, while as $\gamma \rightarrow 1$ the overconfidence becomes negligible and the subjective ability distribution coincides with the objective distribution.

The overconfidence of worker $A$ is common knowledge, all other players know that $A$ is overconfident, while $A$ knows that the other players disagree with his view of his ability distribution. This "agree to disagree" assumption of non-common priors allows us to solve the game in a tractable way.[9]

There are two job levels within each firm: In period 1 (the early career stage), workers are employed by the incumbent firm in the low-level job $L$, but one of them can move to the high-level job $H$ by promotion.[10] Each worker $i$ exerts an effort $e_i \geq e_{\min} > 0$ (where $e_{\min}$ is the minimum effort required to keep the current job) and produces an output equal to

$$y_{i1L} = c_L + d_L e_i \theta_i, \tag{1}$$

where $c_L$ and $d_L$ are strictly positive parameters characterizing the production technology of the low-level job. A higher value of $d_L$ implies a higher sensitivity of output to worker productivity.

By working in period 1, workers acquire two forms of human capital. First, there is firm-

[8] Overestimating one's ability is referred to in the literature as *overoptimism* or *overestimation*, see, e.g., Moore and Healy (2008).

[9] For a discussion of the non-common priors assumption, see, e.g., Savage (1954), Aumann (1976), Kyle and Wang (1997), Brunnermeier and Parker (2005), Santos-Pinto (2010), and Deng et al. (2020). This assumption can be justified in several ways. For example, personality traits are typically revealed during job interviews, in confidential reference letters, or in informal hiring networks.

[10] A worker cannot be hired directly into the high-level job. New workers usually lack the skills required for the high-level job.

specific human capital, characterized by the parameter $S$, which cannot be transferred to another firm. Second, there is transferable human capital acquired through learning-by-doing, $qe_i$, which strictly increases with effort in period 1 and is preserved if the worker leaves the firm. The parameter $q > 0$ captures the relative importance of ability and human capital in determining period-2 productivity.

In period 2 (the late career stage), workers choose the minimum effort, $e_{\min}$, since there are no further incentives in this two-period game. The cost of effort is separable between periods and is given by $c(e_i)$ in period 1 and $c(e_{\min})$ in period 2, where $c'(e) > 0$, $c''(e) > 0$ for all $e > e_{\min}$, and $c'(e_{\min}) = 0$.[11] We assume that, given all other model parameters, the cost function is sufficiently steep such that the equilibrium difference in transferable human capital between workers is less than one, $|q(e_A^* - e_B^*)| < 1$. This assumption rules out the possibility that any worker will be promoted with certainty.

At the end of period 1, one worker is promoted to job $H$ in the incumbent firm and has a period-2 output equal to

$$y_{i2H} = c_H + (1 + S)d_H e_{\min}(\theta_i + qe_i), \tag{2}$$

where $c_H$ and $d_H$ are parameters characterizing the high-level job. The factor $\theta_i + qe_i$ is the period-2 *productivity* of worker $i$, which includes the human capital acquired through learning-by-doing in period 1.

The other worker remains in job $L$ and has a period-2 output of

$$y_{i2L} = c_L + (1 + S)d_L e_{\min}(\theta_i + qe_i). \tag{3}$$

Following Waldman (1984) and others, we assume $c_H < c_L$ and $d_H > d_L$, implying that productivity is more important in the high-level job.[12]

The incumbent firm observes both workers' output in period 1 and promotes a worker to maximize its expected profit. Outside firms cannot observe individual output, but they can observe who has been promoted and use this information to update their assessments of workers' abilities. The external firms simultaneously make individual wage offers to all workers. The incumbent firm observes these offers and makes counteroffers. Each worker is hired by the firm with the highest offer. Ties are broken randomly, except in the case where the period-1 employer is among the firms making the highest offer, in which case a worker remains with the initial employer. It is assumed that firm-specific human capital $S$ is sufficiently high that, in equilibrium, no outside firm succeeds in hiring a worker away from the period-1 employer. Following the literature on promotion signaling (e.g., DeVaro and Waldman, 2012), we assume that there is a small exogenous probability $\tau$ that the incumbent mistakenly fails to make a counteroffer, which is independent of worker ability. This assumption ensures that outside

[11]The cost of effort in the second period is mostly ignored in our analysis because it is constant.
[12]Baker et al. (1994b) argue that higher-level jobs are more sensitive to differences in ability.

firms poach workers with positive probability, implying that the highest equilibrium offer from an outside firm is equal to the worker's expected productivity.[13]

We further assume that external firms always assign workers to the low-level job $L$, regardless of whether the worker was assigned to job $L$ or $H$ by the incumbent firm.[14] If hired by an external firm, the output of worker $i$ would be

$$\hat{y}_{i2L} = c_L + d_L e_{\min}(\theta_i + q e_i). \tag{4}$$

The incumbent firm makes a promotion decision based on expected profit maximization, taking into account the wage offers from outside firms that will be made in response to the promotion decision and that it will have to match to keep the workers.

# 3 Equilibrium Characterization

Proposition 1 describes the incumbent's equilibrium promotion rule as well as the central result that the overconfident worker $A$ exerts more effort than worker $B$.

**Proposition 1.** *In the equilibrium of the promotion game:*

*(a) The worker with the higher period-2 productivity is promoted.*

*(b) The overconfident worker $A$ exerts more effort in period 1 than worker $B$.*

*Proof.* See Online Appendix A.2. □

The intuition behind a) is that, in the absence of commitment power, the only credible promotion rule is one that maximizes the incumbent's expected period-2 profit, given the observed output of the two workers and the expected outside wage offers. This profit is equal to the output produced by the workers minus the wage payments. As the proof of Proposition 1 shows, outside firms offer the same wage premium to both workers upon promotion. This means that the sum of wages for the incumbent (which matches these offers) is constant and independent of who is promoted. This leaves output as the decisive criterion for the promotion decision. As can be seen in (2) and (3), output depends crucially on productivity $\theta_i + q e_i$. By the assumptions $c_H < c_L$ and $d_H > d_L$, promoting the worker with higher productivity is the profit-maximizing decision.

[13] If this assumption were dropped, the same equilibrium would exist where the highest equilibrium offer from an outside firm equals the worker's expected productivity. However, the equilibrium would not be unique, and other outcomes of period-2 bargaining would be possible. Furthermore, the assumption that $\tau$ is independent of worker ability eliminates the strong winner's curse result that occurs in other asymmetric learning models with firm-specific human capital and counteroffers (e.g., Ghosh and Waldman, 2010, DeVaro and Waldman, 2012, Cassidy et al., 2016, and Waldman and Zax, 2016).

[14] All of our qualitative results would be the same if external firms always assigned workers to the high-level job $H$.

The intuition behind b) can be seen as follows. Since ability realizations $\theta_i$ and effort levels $e_i$ are not observable, the incumbent firm forms beliefs about these variables based on the observable output $y_{i1L}$, and these beliefs are denoted by $\tilde{\theta}_i$ and $\tilde{e}_i$. Based on equation (1), $\tilde{\theta}_i$ is derived from the observed output $y_{i1L}$ and $\tilde{e}_i$ as follows:

$$\tilde{\theta}_i = \frac{y_{i1L} - c_L}{d_L \tilde{e}_i}. \tag{5}$$

The incumbent's effort belief $\tilde{e}_i$ is taken as given by the workers, since they cannot affect it by their (unobservable) effort choice $e_i$. The incumbent's ability belief $\tilde{\theta}_i$ is independent of the ability distribution (overconfident or not), since it is a belief about ability *realization* derived from actual observed performance. Thus, the only way for a worker to affect the probability of promotion is to change output by changing effort. In equilibrium, $\tilde{e}_i$ is equal to the actual effort $e_i$ chosen by worker $i$, and therefore $\tilde{\theta}_i$ is equal to the actual ability realization $\theta_i$ of worker $i$. Since effort and ability are complements in producing output, the overconfident worker $A$ who overestimates his expected ability mistakenly believes that his effort is marginally more effective at increasing output than it actually is, motivating $A$ to choose a higher effort. In turn, $A$'s higher effort, and thus the probability of promotion, discourages player $B$ by making a given effort $e_B$ less effective for promotion.

Our second result concerns the differences in outcomes between the two workers. These are a direct consequence of the higher effort exerted by the overconfident worker $A$.

**Corollary 1.** *In equilibrium, compared to worker $B$, the overconfident worker $A$*

*(a) Is promoted with a higher probability.*

*(b) Receives a higher period-2 wage conditional on the job level.*

*(c) Receives a higher expected period-2 wage.*

*(d) Has a lower expected ability conditional on promotion.*

*(e) Acquires more transferable human capital through learning-by-doing.*

*Proof.* See Online Appendix A.3. □

Part (a) is easy to see: Given that worker $A$ exerts more effort than $B$, while the ability of both workers is drawn from the same distribution, the promotion rule that in equilibrium compares the productivity of $\theta_A + qe_A$ and $\theta_B + qe_B$ will select $A$ more often.

Part (b) is the result of two opposing effects. Outside firms only care about and pay for a worker's productivity $\theta_i + qe_i^*$. Due to the higher effort, the transferable human capital of $A$, $qe_A^*$, exceeds that of worker $B$, while, see part (d), the conditional expected ability of $B$, $\theta_B$, exceeds that of $A$. We prove that the overall effect is unambiguously in favor of $A$.

Part (c) follows directly from the combination of a higher probability of promotion, part (a), with higher wages conditional on promotion, part (b).

To understand part (d), recall that the promotion rule compares productivity $\theta_A + qe_A$ with $\theta_B + qe_B$, and selects the more more productive worker. Thus, for worker $B$ to be promoted, it must hold that $\theta_B > \theta_A + q(e_A - e_B)$. This means that $B$'s ability must exceed both $A$'s ability and $A$'s advantage due to the higher effort. In contrast, the promotion of $A$ requires $\theta_A > \theta_B - q(e_A - e_B)$, meaning that $A$ can be promoted even if $\theta_A$ is slightly below $\theta_B$. Overall, this makes it more difficult for $B$ to be promoted than for $A$, which means that a promoted $B$ tends to have greater ability than a promoted $A$.

Part e) follows directly from the fact that effort is higher for the overconfident worker.

We present two numerical examples that illustrate all of our key results. Consider the following parameters:

$$\gamma = 2, c_L = 2,\ c_H = 1,\ d_L = 1,\ d_H = 2,\ c(e) = \frac{(e - e_{\min})^2}{2},\ e_{\min} = \frac{1}{5},\ q = 2.$$

In the first example, shown in Table 1, the expected utility of worker $A$ exceeds that of worker $B$, and it is also higher than it would be in a game in which neither worker is overconfident. Thus, in this example, overconfidence is a self-serving bias.

Table 1: Numerical Example, $q = 2$.

| | worker $A$ | worker $B$ |
|---|---|---|
| equilibrium effort | 0.324 | 0.308 |
| promotion probability | 0.532 | 0.468 |
| effort cost | 0.0077 | 0.0058 |
| expected period-2 wage offer if promoted | 2.261 | 2.259 |
| expected period-2 wage offer if not promoted | 2.194 | 2.192 |
| expected utility | 2.222 | 2.217 |
| expected ability conditional on promotion | 0.656 | 0.677 |
| expected productivity conditional on promotion | 1.305 | 1.294 |
| $A$'s subjective promotion probability | 0.698 | |
| $A$'s subjective expected utility | 2.233 | |

Now we change $q = 2$ to $q = 1/2$, which makes human capital formation less sensitive to effort, thus reducing the advantage of $A$ due to higher effort. All other parameters remain unchanged. The results, shown in Table 2, show that the expected utility of $B$ now exceeds that of $A$. Overconfidence becomes self-defeating, as $A$'s expected utility is now lower than it would be in a game without overconfidence.

Table 2: Numerical Example, $q = 1/2$.

| | worker $A$ | worker $B$ |
|---|---|---|
| equilibrium effort | 0.330 | 0.308 |
| promotion probability | 0.511 | 0.489 |
| effort cost | 0.0085 | 0.0058 |
| expected period-2 wage offer if promoted | 2.166 | 2.165 |
| expected period-2 wage offer if not promoted | 2.099 | 2.098 |
| expected utility | 2.124 | 2.125 |
| expected ability conditional on promotion | 0.663 | 0.670 |
| expected productivity conditional on promotion | 0.828 | 0.824 |
| $A$'s subjective promotion probability | 0.678 | |
| $A$'s subjective expected utility | 2.136 | |

# 4 Policy Implications

Proposition 1 showed that the gender differences in labor market outcomes in our setting are all driven by the higher effort of the overconfident worker. This suggests that policies that limit working hours can mitigate the effects of overconfidence on labor market outcomes. Proposition 2 below shows that an appropriate upper bound on effort would completely eliminate any gender inequality in the labor market driven by overconfidence.[15]

**Proposition 2.** *Suppose a regulator imposes an upper bound on effort, $\bar{e} > 0$, equal to the symmetric equilibrium effort $\hat{e}$ obtained in the absence of overconfidence. For sufficiently steep cost functions, the modified game has an equilibrium in which both workers choose effort $\bar{e}$. All labor market outcomes become fully symmetric and equal to the outcomes obtained in the absence of overconfidence. In particular, all workers have the same probability of promotion and the same wage conditional on promotion.*

*Proof.* See Online Appendix A.7. □

In reality, the exact enforcement of such a limit may be difficult, especially in highly skilled occupations. In many modern labor markets, however, regulation of working hours is commonplace. For example, the Swedish "Working Hours Act" (*arbetstidslag*) explicitly aims to protect workers from working too much and sets limits on daily, weekly and annual working hours.[16] The motivation behind such policies is typically to promote worker health and reduce

[15] In our numerical examples, the (quadratic) cost function is sufficiently steep in the sense of the proposition. Moreover, in the equilibrium of the game with the effort constraint, the constraint is binding for the overconfident worker $A$, while worker $B$ plays her unconstrained best response.

[16] For details, see https://www.av.se/en/work-environment-work-and-inspections/acts-and-regulations-about-work-environment/the-working-hours-act/the-working-hours-act-in-brief/.

the risk of burnout. Our results suggest that such policies may also be conducive to promoting gender equality in the labor market.[17] Of course, these gains in gender equality would have to be weighed against the associated efficiency consequences.

There are also firm-level policies that can be used to influence the confidence levels of workers. Deng et al. (2020) study theoretically the optimal information disclosure policy of a firm employing over- or underconfident workers. They characterize the conditions under which de-biasing can be in the firm's interest or detrimental to the firm's performance. The empirical literature documents mixed evidence on the success of various interventions to address overconfidence, see Hügelschäfer and Achtziger (2014) for a comprehensive discussion of the empirical evidence.[18]

# 5 Concluding Remarks

Recent literature examining the "last chapter" of gender inequality in the labor market has pointed to the ways in which firms reward long consecutive (and "particular") hours of work, as well as the key role that certain psychological traits and non-cognitive skills play in predicting competitive behavior (Goldin, 2014). In this paper, we have examined how male overconfidence, combined with competitive incentives in the workplace, affects gender equality in the labor market. The framework of analysis has been a promotion signaling model, in which wages are realistically shaped by market forces. We have also briefly discussed policy implications in terms of labor market regulation and firm-level confidence management.

There are certain aspects that our model did not cover. First, we considered competition for promotion among workers with the same prior education. While this approach is clearly relevant (e.g., Azmat and Ferrer, 2017), it does not take into account how the labor market effects of overconfidence (in terms of promotion probabilities and wages) would affect the selection of people into demanding educational tracks or occupations that would expose them to competitive wages, which Blau and Kahn (2017) identifies as important drivers of the remaining gender gap. Second, for tractability reasons, we have not examined how risk aversion would interact with the different confidence levels in our setup. Third, we did not analyze the division of childcare responsibilities. Fourth, we assumed that all work effort is productive, whereas in reality workers engage in a combination of productive and rent-seeking effort. We hope that these questions will attract further research in the future.

[17] In this way, from a gender equality perspective, such policies serve as a complement to other policies that allow workers to combine work and family life, such as subsidized child care and parental leave arrangements, see for example Bastani et al. (2019, 2020).

[18] Most empirical studies are conducted using laboratory experiments. For example, Grossman and Owens (2012) found that overconfidence in *own* ability is difficult to influence by interventions, whereas Chen and Schildberg-Hörisch (2019) showed that de-biasing, in the form of providing information about ability, reduces overconfidence-driven effort.

# References


Acemoglu, D. and Pischke, J.-S. (1998). Why do firms train? theory and evidence. *Quarterly Journal of Economics*, 113(1):79–119.

Adamecz-Völgyi, A. and Shure, N. (2022). The gender gap in top jobs – the role of overconfidence. *Labour Economics*, 79:102283.

Aumann, R. J. (1976). Agreeing to disagree. *Annals of statistics*, 4(6):1236–1239.

Azmat, G. and Ferrer, R. (2017). Gender gaps in performance: Evidence from young lawyers. *Journal of Political Economy*, 125(5):1306–1355.

Babcock, L., Recalde, M. P., Vesterlund, L., and Weingart, L. (2017). Gender differences in accepting and receiving requests for tasks with low promotability. *American Economic Review*, 107(3):714–747.

Baker, G., Gibbs, M., and Holmström, B. (1994a). The internal economics of the firm: Evidence from personnel data. *Quarterly Journal of Economics*, 109(4):881–919.

Baker, G., Gibbs, M., and Holmström, B. (1994b). The wage policy of a firm. *The Quarterly Journal of Economics*, 109(4):921–955.

Bastani, S., Blomquist, S., and Micheletto, L. (2020). Child care subsidies, quality, and optimal income taxation. *American Economic Journal: Economic Policy*, 12(4):1–37.

Bastani, S., Blumkin, T., and Micheletto, L. (2019). The welfare-enhancing role of parental leave mandates. *Journal of Law, Economics, and Organization*, 35(1):77–126.

Bénabou, R. and Tirole, J. (2002). Self-confidence and personal motivation. *Quarterly Journal of Economics*, 117(3):871–915.

Bender, S., Bloom, N., Card, D., Van Reenen, J., and Wolter, S. (2018). Management practices, workforce selection, and productivity. *Journal of Labor Economics*, 36(S1):S371–S409.

Bernhardt, D. (1995). Strategic promotion and compensation. *Review of Economic Studies*, 62(2):315–339.

Bertrand, M., Goldin, C., and Katz, L. F. (2010). Dynamics of the gender gap for young professionals in the financial and corporate sectors. *American Economic Journal: Applied Economics*, 2(3):228–55.

Blau, F. D. and Kahn, L. M. (2017). The gender wage gap: Extent, trends, and explanations. *Journal of Economic Literature*, 55(3):789–865.

Bognanno, M. and Melero, E. (2016). Promotion signals, experience, and education. *Journal of Economics & Management Strategy*, 25(1):111–132.

Bognanno, M. L. (2001). Corporate tournaments. *Journal of Labor Economics*, 19(2):290–315.

Bosquet, C., Combes, P.-P., et al. (2019). Gender and promotions: evidence from academic economists in france. *Scandinavian Journal of Economics*, 121(3):1020–1053.

Bruhin, A., Petros, F., and Santos-Pinto, L. (2022). The role of self-confidence in teamwork: Experimental evidence. *Working paper*.

Brunnermeier, M. K. and Parker, J. A. (2005). Optimal expectations. *American Economic Review*, 95(4):1092–1118.

Camerer, C. and Lovallo, D. (1999). Overconfidence and excess entry: An experimental approach. *The American Economic Review*, 89(1):306–318.

Campbell, E. L. and Hahl, O. (2022). He's overqualified, she's highly committed: Qualification signals and gendered assumptions about job candidate commitment. *Organization Science*, 33(6):2451–2476.

Caplin, A., Lee, M., Leth-Petersen, S., Saeverud, J., and Shapiro, M. D. (2022). How worker productivity and wages grow with tenure and experience: The firm perspective. *NBER Working Paper No. 30342*, (30342).

Cassidy, H., DeVaro, J., and Kauhanen, A. (2016). Promotion signaling, gender, and turnover: New theory and evidence. *Journal of Economic Behavior & Organization*, 126:140–166.

Chen, S. and Schildberg-Hörisch, H. (2019). Looking at the bright side: The motivational value of confidence. *European Economic Review*, 120:103302.

Cook, C., Diamond, R., Hall, J. V., List, J. A., and Oyer, P. (2021). The gender earnings gap in the gig economy: Evidence from over a million rideshare drivers. *Review of Economic Studies*, 88(5):2210–2238.

Croson, R. and Gneezy, U. (2009). Gender differences in preferences. *Journal of Economic Literature*, 47(2):1–27.

Dato, S. and Nieken, P. (2014). Gender differences in competition and sabotage. *Journal of Economic Behavior & Organization*, 100:64–80.

De Grip, A., Sauermann, J., and Sieben, I. (2016). The role of peers in estimating tenure-performance profiles: Evidence from personnel data. *Journal of Economic Behavior & Organization*, 126:39–54.

Deng, S., Fang, H., Fu, Q., and Wu, Z. (2020). Confidence management in tournaments. *NBER Working Paper No. 27186*.

DeVaro, J. (2006). Strategic promotion tournaments and worker performance. *Strategic Management Journal*, 27(8):721–740.

DeVaro, J., Ghosh, S., and Zoghi, C. (2018). Job characteristics and labor market discrimination in promotions. *Industrial Relations*, 57(3):389–434.

DeVaro, J., Kauhanen, A., and Valmari, N. (2019). Internal and external hiring. *ILR Review*, 72(4):981–1008.

DeVaro, J. and Waldman, M. (2012). The signaling role of promotions: Further theory and empirical evidence. *Journal of Labor Economics*, 30(1):91–147.

Dreber, A., von Essen, E., and Ranehill, E. (2014). Gender and competition in adolescence: task matters. *Experimental Economics*, 17(1):154–172.

Flory, J., Leibbrandt, A., Rott, C., and Stoddard, O. (2021). Signals from on high and the power of growth mindset: A natural field experiment in attracting minorities to high-profile positions. *IZA Discussion Paper No. 14383*.

Flory, J. A., Leibbrandt, A., and List, J. A. (2015). Do competitive workplaces deter female workers? a large-scale natural field experiment on job entry decisions. *Review of Economic Studies*, 82(1):122–155.

Ghosh, S. and Waldman, M. (2010). Standard promotion practices versus up-or-out contracts. *RAND Journal of Economics*, 41(2):301–325.

Gneezy, U., Niederle, M., and Rustichini, A. (2003). Performance in competitive environments: Gender differences. *Quarterly Journal of Economics*, 118(3):1049–1074.

Goldin, C. (2014). A grand gender convergence: Its last chapter. *American Economic Review*, 104(4):1091–1119.

Green, J. R. and Stokey, N. L. (1983). A comparison of tournaments and contracts. *Journal of Political Economy*, 91(3):349–364.

Grossman, Z. and Owens, D. (2012). An unlucky feeling: Overconfidence and noisy feedback. *Journal of Economic Behavior & Organization*, 84(2):510–524.

Gürtler, M. and Gürtler, O. (2015). The optimality of heterogeneous tournaments. *Journal of Labor Economics*, 33(4):1007–1042.

Gürtler, M. and Gürtler, O. (2019). Promotion signaling, discrimination, and positive discrimination policies. *RAND Journal of Economics*, 50(4):1004–1027.

Heyman, F., Norbäck, P.-J., and Persson, L. (2020). Talent, career choice and competition: The gender wage gap at the top. *CESifo Working Paper No. 8657*.

Hoffman, M. and Burks, S. V. (2020). Worker overconfidence: Field evidence and implications for employee turnover and firm profits. *Quantitative Economics*, 11(1):315–348.

Holmström, B. (1982). Moral hazard in teams. *Bell Journal of Economics*, 13:324–340.

Hügelschäfer, S. and Achtziger, A. (2014). On confident men and rational women: It's all on your mind(set). *Journal of Economic Psychology*, 41:31–44. From Dual Processes to Multiple Selves: Implications for Economic Behavior.

James, S., Lagerqvist, B., Lundborg, P., and Vikström, J. (2022). Learning-by-doing and productivity growth among high-skilled workers: Evidence from the treatment of heart attacks. *IFAU Working Paper No. 2022:3*.

Keloharju, M., Knüpfer, S., and Tåg, J. (2016). Equal opportunity? gender gaps in ceo appointments and executive pay. *IFN Working Paper No. 1111*.

Keloharju, M., Knüpfer, S., and Tåg, J. (2022). What prevents women from reaching the top? *Financial Management*.

Kyle, A. S. and Wang, F. A. (1997). Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? *Journal of Finance*, 52(5):2073–2090.

Landers, R. M., Rebitzer, J. B., and Taylor, L. J. (1996). Rat race redux: Adverse selection in the determination of work hours in law firms. *The American Economic Review*, 86(3):329–348.

Lazear, E. P. and Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy*, 89(5):841–864.

Malcomson, J. M. (1984). Work incentives, hierarchy, and internal labor markets. *Journal of Political Economy*, 92(3):486–507.

Manning, A. and Swaffield, J. (2008). The Gender Gap in Early-Career Wage Growth. *The Economic Journal*, 118(530):983–1024.

Moore, D. A. and Healy, P. J. (2008). The trouble with overconfidence. *Psychological Review*, 115(2):502–517.

Niederle, M. and Vesterlund, L. (2007). Do women shy away from competition? do men compete too much? *Quarterly Journal of Economics*, 122(3):1067–1101.

Noonan, M. C., Corcoran, M. E., and Courant, P. N. (2005). Pay Differences Among the Highly Trained: Cohort Differences in the Sex Gap in Lawyers' Earnings. *Social Forces*, 84(2):853–872.

Prendergast, C. (1999). The provision of incentives in firms. *Journal of Economic Literature*, 37(1):7–63.

Santos-Pinto, L. (2010). Positive self-image in tournaments. *International Economic Review*, 51(2):475–496.

Santos-Pinto, L. (2021). Are overconfident players more likely to win tournaments and contests? *Working paper*.

Santos-Pinto, L. (2022). Are overconfident workers more likely to win bonuses or be promoted? *Discussion Paper*.

Santos-Pinto, L. and Sekeris, P. (2022). Overconfidence in tullock contests. *working paper*.

Sarsons, H. and Xu, G. (2021). Confidence men? evidence on confidence and gender among top economists. *AEA Papers and Proceedings*, 111:65–68.

Savage, L. J. (1954). *The foundations of statistics*. John Wiley, New York.

Säve-Söderbergh, J. (2019). Gender gaps in salary negotiations: Salary requests and starting salaries in the field. *Journal of Economic Behavior & Organization*, 161:35–51.

Stinebrickner, R., Stinebrickner, T., and Sullivan, P. (2019). Job tasks, time allocation, and wages. *Journal of Labor Economics*, 37(2):399–433.

van Veldhuizen, R. (2022). Gender differences in tournament choices: Risk preferences, overconfidence or competitiveness? *Journal of the European Economic Association*. jvac031.

Waldman, M. (1984). Job assignments, signalling, and efficiency. *RAND Journal of Economics*, 15(2):255–267.

Waldman, M. (1990). Up-or-out contracts: A signaling perspective. *Journal of Labor Economics*, 8(2):230–250.

Waldman, M. (1994). Systematic errors and the theory of natural selection. *The American economic review*, 84(3):482–497.

Waldman, M. (2013). Classic promotion tournaments versus market-based tournaments. *International Journal of Industrial Organization*, 31(3):198–210.

Waldman, M. (2016). The dual avenues of labor market signaling. *Labour Economics*, 41:120–134.

Waldman, M. and Zax, O. (2016). An exploration of the promotion signaling distortion. *Journal of Law, Economics, and Organization*, 32(1):119–149.

Zábojník, J. (2012). Promotion tournaments in market equilibrium. *Economic Theory*, 51(1):213–240.

Zábojník, J. and Bernhardt, D. (2001). Corporate Tournaments, Human Capital Acquisition, and the Firm Size—Wage Relation. *Review of Economic Studies*, 68(3):693–716.

Zimmermann, F. (2020). The dynamics of motivated beliefs. *American Economic Review*, 110(2):337–61.

# Supplementary Material (Online Appendix)

## A Proofs and derivations

### A.1 Preliminary results

We start by proving a set of preliminary results to be used in the proofs of our main results. Throughout the appendix, we make use of the random variables $\Theta_A$ and $\Theta_B$ that are assumed to be uniformly distributed on $[0,1]$ with the following pdf and cdf

$$f(x) = \begin{cases} 1 & x \in [0,1] \\ 0 & \text{else} \end{cases}, \qquad F(x) = \begin{cases} 0 & x < 0 \\ x & x \in [0,1] \\ 1 & x > 1 \end{cases}. \tag{A1}$$

We also make use of the random variable $\hat{\Theta}_A$ with support $[0,1]$ and the following pdf and cdf, where $\gamma > 1$.

$$\hat{f}(x) = \begin{cases} \gamma x^{\gamma-1} & x \in [0,1] \\ 0 & \text{else,} \end{cases} \qquad \hat{F}(x) = \begin{cases} 0 & x < 0 \\ x^{\gamma} & x \in [0,1] \\ 1 & x > 1. \end{cases} \tag{A2}$$

The following lemma computes probabilities that will later be shown to be the equilibrium subjective, resp. objective, promotion probabilities of worker $A$ (case (a), resp. (b)), and the (objective) promotion probability of worker $B$ (case (c)).

**Lemma 1.** *For a constant $K \in (0,1)$, we have the following probabilities.*

$$(a) \qquad \hat{P}(\hat{\Theta}_A + K > \Theta_B) := \int_{-\infty}^{\infty} F(x+K)\, \hat{f}(x)\, dx = 1 - \frac{(1-K)^{\gamma+1}}{\gamma+1} \tag{A3}$$

$$(b) \qquad P(\Theta_A + K > \Theta_B) := \int_{-\infty}^{\infty} F(x+K)\, f(x)\, dx = \frac{1}{2}\left(1 + 2K - K^2\right) \tag{A4}$$

$$(c) \qquad P(\Theta_A + K < \Theta_B) := \int_{-\infty}^{\infty} F(x-K)\, f(x)\, dx = \frac{1}{2}(1-K)^2. \tag{A5}$$

*Proof of Lemma 1.* (a)

$$
\begin{aligned}
\int_{-\infty}^{\infty} F(x+K)\,\hat{f}(x)\,dx &= \gamma \int_0^1 F(x+K)\,x^{\gamma-1}dx \\
&= \gamma\left(\int_0^{1-K}\left(x^{\gamma}+Kx^{\gamma-1}\right)dx + \int_{1-K}^{1} x^{\gamma-1}dx\right) \\
&= \gamma\left(\frac{1}{\gamma+1}(1-K)^{\gamma+1} + \frac{K}{\gamma}(1-K)^{\gamma} + \frac{1}{\gamma} - \frac{1}{\gamma}(1-K)^{\gamma}\right) \\
&= 1 + \gamma\left(\frac{1}{\gamma+1}(1-K)^{\gamma+1} - \frac{(1-K)^{\gamma}}{\gamma}(1-K)\right) \\
&= 1 + (1-K)^{\gamma+1}\left(\frac{\gamma}{\gamma+1} - 1\right) \\
&= 1 - \frac{(1-K)^{\gamma+1}}{\gamma+1}.
\end{aligned}
\tag{A6}
$$

(b)

$$
\begin{aligned}
\int_{-\infty}^{\infty} F(x+K)f(x)dx &= \int_0^1 F(x+K)dx \\
&= \int_0^{1-K}(x+K)dx + \int_{1-K}^{1} 1dx \\
&= \frac{1}{2}(1+2K-K^2).
\end{aligned}
\tag{A7}
$$

(c) This can be computed directly from (b):

$$
\begin{aligned}
P(\Theta_A + K < \Theta_B) &= 1 - P(\Theta_A + K > \Theta_B) \\
&= 1 - \frac{1}{2}(1+2K-K^2) \\
&= \frac{1}{2}(1-2K+K^2).
\end{aligned}
\tag{A8}
$$

□

**Lemma 2.** *For any constant $K \in (0,1)$, we have*

$$(a) \qquad E[\Theta_A|\Theta_A + K > \Theta_B] = \frac{2+3K-3K^2+K^3}{3(1+2K-K^2)}, \qquad \text{(A9)}$$

$$(b) \qquad E[\Theta_A|\Theta_A + K < \Theta_B] = \frac{1-K}{3}, \qquad \text{(A10)}$$

$$(c) \qquad E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B] = \frac{1+2K}{3(1+2K-K^2)}, \qquad \text{(A11)}$$

$$(d) \qquad E[\Theta_B|\Theta_A + K > \Theta_B] = \frac{1+3K-K^3}{3(1+2K-K^2)}, \qquad \text{(A12)}$$

$$(e) \qquad E[\Theta_B|\Theta_A + K < \Theta_B] = \frac{2+K}{3}. \qquad \text{(A13)}$$

*Proof of Lemma 2.* (a) As both random variables are uniformly distributed on $[0,1]$ and $P\left(\Theta_B < \Theta_A + K\right) = \frac{1}{2}(1+2K-K^2)$, as shown in (A4), we obtain

$$\begin{aligned}
E\left[\Theta_A \middle| \Theta_A + K > \Theta_B\right] &= E\left[\Theta_A \middle| \Theta_A > \Theta_B - K\right] \\
&= \frac{\int_0^1 \int_{\max\{y-K,0\}}^1 x dx dy}{\frac{1}{2}(1+2K-K^2)} \\
&= \frac{\int_K^1 \int_{y-K}^1 x dx dy + \int_0^K \int_0^1 x dx dy}{\frac{1}{2}(1+2K-K^2)} \\
&= \frac{\frac{1}{2}\int_K^1 \left(1-y^2+2Ky-K^2\right) dy + \frac{1}{2}\int_0^K dy}{\frac{1}{2}(1+2K-K^2)} \\
&= \frac{3\left(-\frac{1}{3}\left(1-K^3\right) + K\left(1-K^2\right) + \left(1-K^2\right)\left(1-K\right)\right) + 3K}{3+6K-3K^2} \\
&= \frac{2+3K-3K^2+K^3}{3+6K-3K^2} \\
&= \frac{K\left(\left(K-3\right)K+3\right)+2}{3-3\left(K-2\right)K}.
\end{aligned}$$

(b) As $P\left(\Theta_B > \Theta_A + K\right) = \frac{1}{2}(1-K)^2$, as shown in (A5), we obtain

$$
\begin{aligned}
E\left[\Theta_A \middle| \Theta_A + K < \Theta_B\right] &= E\left[\Theta_A \middle| \Theta_A < \Theta_B - K\right] \\
&= \frac{\int_K^1 \int_0^{y-K} x dx dy}{\frac{1}{2}(1-K)^2} \\
&= \frac{\frac{1}{2}\int_K^1 (y-K)^2 dy}{\frac{1}{2}(1-K)^2} \\
&= \frac{\frac{1}{3}\left(1-K^3\right) - K\left(1-K^2\right) + K^2\left(1-K\right)}{\left(1-K\right)^2} \\
&= \frac{1 - K^3 - 3K + 3K^2}{3\left(1-K\right)^2} \\
&= \frac{\left(1-K\right)^3}{3\left(1-K\right)^2} \\
&= \frac{1-K}{3}.
\end{aligned}
$$

(c) It directly follows that

$$
\begin{aligned}
&E\left[\Theta_A \middle| \Theta_A + K > \Theta_B\right] - E\left[\Theta_A \middle| \Theta_A < \Theta_B - K\right] \\
&= \frac{2 + 3K - 3K^2 + K^3}{3 + 6K - 3K^2} - \frac{1-K}{3} \\
&= \frac{2 + 3K - 3K^2 + K^3 - (1-K)\left(1 + 2K - K^2\right)}{3 + 6K - 3K^2} \\
&= \frac{1 + 2K}{3(1 + 2K - K^2)}.
\end{aligned}
$$

(d) As both random variables are uniformly distributed on $[0,1]$ and $P\left(\Theta_B < \Theta_A + K\right) = \frac{1}{2}(1 + 2K - K^2)$, as shown in (A4), we obtain

$$
\begin{aligned}
E\left[\Theta_B \middle| \Theta_B < \Theta_A + K\right] &= \frac{\int_0^1 \int_0^{\min\{1, y+K\}} x dx dy}{\frac{1}{2}(1 + 2K - K^2)} \\
&= \frac{\int_0^{1-K} \int_0^{y+K} x dx dy + \int_{1-K}^1 \int_0^1 x dx dy}{\frac{1}{2}(1 + 2K - K^2)} \\
&= \frac{\frac{1}{2}\int_0^{1-K} (y^2 + 2Ky + K^2) dy + \frac{1}{2}\int_{1-K}^1 dy}{\frac{1}{2}(1 + 2K - K^2)} \\
&= \frac{\frac{1}{2}\left(\frac{1}{3}\left(1-K\right)^3 + K\left(1-K\right)^2 + K^2\left(1-K\right)\right) + \frac{K}{2}}{\frac{1}{2}(1 + 2K - K^2)} \\
&= \frac{1 + 3K - K^3}{3(1 + 2K - K^2)}.
\end{aligned}
$$

(e) As $P\left(\Theta_B > \Theta_A + K\right) = \frac{1}{2}(1 - 2K + K^2)$, as shown in (A5), we obtain

$$\begin{aligned}
E\left[\Theta_B | \Theta_B > \Theta_A + K\right] &= \frac{\int_0^{1-K}\int_{y+K}^{1} x dx dy}{\frac{1}{2}(1-2K+K^2)} \\
&= \frac{\frac{1}{2}\int_0^{1-K}\left(1 - y^2 - 2Ky - K^2\right) dy}{\frac{1}{2}(1-2K+K^2)} \\
&= \frac{1 - K - \frac{1}{3}\left(1-K\right)^3 - K\left(1-K\right)^2 - K^2\left(1-K\right)}{1-2K+K^2} \\
&= \frac{2 - 3K + K^3}{3\left(1-K\right)^2} \\
&= \frac{\left(2+K\right)\left(1-K\right)^2}{3\left(1-K\right)^2} \\
&= \frac{2+K}{3}.
\end{aligned}$$

□

**Lemma 3.** *For a constant $K \in (-1, 1)$, and $f$ and $\hat{f}$ defined in* (A1) *and* (A2)*, we have*

$$(a) \qquad \int_{-\infty}^{\infty} f(x+K)x\hat{f}(x)dx = \begin{cases} \frac{\gamma}{1+\gamma}(1 + K(-K)^{\gamma}) & -1 < K \leq 0 \\ \frac{\gamma}{1+\gamma}(1-K)^{1+\gamma} & 0 < K < 1, \end{cases} \tag{A14}$$

$$(b) \qquad \int_{-\infty}^{\infty} f(x-K)xf(x)dx = \begin{cases} \frac{1}{2}(1+K)^2 & -1 < K \leq 0 \\ \frac{1}{2}(1-K^2) & 0 < K < 1. \end{cases} \tag{A15}$$

*Proof of Lemma 3.* (a)

$$\begin{aligned}
\int_{-\infty}^{\infty} f\left(x+K\right) x \hat{f}\left(x\right) dx &= \begin{cases} \gamma \int_{-K}^{1} x^{\gamma} dx & -1 < K \leq 0 \\ \gamma \int_{0}^{1-K} x^{\gamma} dx & 0 < K < 1, \end{cases} \\
&= \begin{cases} \frac{\gamma}{\gamma+1}\left(1 + K(-K)^{\gamma}\right) & -1 < K \leq 0 \\ \frac{\gamma}{\gamma+1}\left(1-K\right)^{\gamma+1} & 0 < K < 1. \end{cases}
\end{aligned}$$

(b)

$$\begin{aligned}
\int_{-\infty}^{\infty} f\left(x-K\right) x f\left(x\right) dx &= \begin{cases} \int_{0}^{1+K} x dx & -1 < K \leq 0 \\ \int_{K}^{1} x dx & 0 < K < 1, \end{cases} \\
&= \begin{cases} \frac{1}{2}(1+K)^2 & -1 < K \leq 0 \\ \frac{1}{2}\left(1-K^2\right) & 0 < K < 1. \end{cases}
\end{aligned}$$

□

**Lemma 4.** *For the constants $e_A, e_A^* > 0$, $\gamma > 1$, and $K \in (0,1)$ we have*

$$(a) \qquad \int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*}+K\right)\hat{f}(x)\,dx = \begin{cases} \frac{\gamma}{1+\gamma}\frac{e_A}{e_A^*}+K & \frac{e_A}{e_A^*}+K \le 1 \\ 1-\frac{(1-K)^{1+\gamma}}{1+\gamma}\left(\frac{e_A^*}{e_A}\right)^{\gamma} & \frac{e_A}{e_A^*}+K > 1, \end{cases} \tag{A16}$$

$$(b) \qquad \frac{\partial}{\partial e_A}\left(\int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*}+K\right)\hat{f}(x)\,dx\right) = \begin{cases} \frac{\gamma}{(1+\gamma)e_A^*} & \frac{e_A}{e_A^*}+K \le 1 \\ \frac{(1-K)^{1+\gamma}}{e_A}\frac{\gamma}{1+\gamma}\left(\frac{e_A^*}{e_A}\right)^{\gamma} & \frac{e_A}{e_A^*}+K > 1, \end{cases} \tag{A17}$$

$$(c) \qquad \frac{\partial^2}{(\partial e_A)^2}\left(\int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*}+K\right)\hat{f}(x)\,dx\right) = \begin{cases} 0 & \frac{e_A}{e_A^*}+K \le 1 \\ -\frac{\gamma(1-K)^{1+\gamma}}{e_A^2}\left(\frac{e_A^*}{e_A}\right)^{\gamma} & \frac{e_A}{e_A^*}+K > 1. \end{cases} \tag{A18}$$

*Proof of Lemma 4.* (a) We want to determine

$$\int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*}+K\right)\hat{f}(x)\,dx = \int_0^1 F\left(x\frac{e_A}{e_A^*}+K\right)\gamma x^{\gamma-1}dx.$$

Consider the term $F\left(x\frac{e_A}{e_A^*}+K\right)$ on the RHS of the above, and note that the argument is strictly positive. If $\frac{e_A}{e_A^*}+K \le 1$, then $F\left(x\frac{e_A}{e_A^*}+K\right) = x\frac{e_A}{e_A^*}+K$, whereas if $\frac{e_A}{e_A^*}+K > 1$, then

$$F\left(x\frac{e_A}{e_A^*}+K\right) = \begin{cases} x\frac{e_A}{e_A^*}+K & x \le (1-K)\frac{e_A^*}{e_A} \\ 1 & x > (1-K)\frac{e_A^*}{e_A}. \end{cases}$$

Therefore,

$$\begin{aligned}\int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*}+K\right)\hat{f}(x)\,dx &= \gamma\int_0^1 x^{\gamma-1}F\left(x\frac{e_A}{e_A^*}+K\right)dx \\ &= \begin{cases} \gamma\int_0^1 x^{\gamma-1}\left(x\frac{e_A}{e_A^*}+K\right)dx & \frac{e_A}{e_A^*}+K \le 1 \\ \gamma\int_0^{(1-K)\frac{e_A^*}{e_A}} x^{\gamma-1}\left(x\frac{e_A}{e_A^*}+K\right)dx + \gamma\int_{(1-K)\frac{e_A^*}{e_A}}^1 x^{\gamma-1}dx & \frac{e_A}{e_A^*}+K > 1. \end{cases}\end{aligned}$$

It is straightforward to compute

$$\gamma \int_0^1 x^{\gamma-1}\left(x\frac{e_A}{e_A^*}+K\right)dx = \gamma \int_0^1 x^{\gamma}\frac{e_A}{e_A^*}+x^{\gamma-1}Kdx = \frac{\gamma}{1+\gamma}\frac{e_A}{e_A^*}+K,$$

$$\gamma \int_0^{(1-K)\frac{e_A^*}{e_A}} x^{\gamma-1}\left(x\frac{e_A}{e_A^*}+K\right)dx = \gamma \int_0^{(1-K)\frac{e_A^*}{e_A}} x^{\gamma}\frac{e_A}{e_A^*}+x^{\gamma-1}Kdx$$

$$= \frac{\gamma}{1+\gamma}\left((1-K)\frac{e_A^*}{e_A}\right)^{\gamma+1}\frac{e_A}{e_A^*}+\left((1-K)\frac{e_A^*}{e_A}\right)^{\gamma}K,$$

$$\gamma \int_{(1-K)\frac{e_A^*}{e_A}}^1 x^{\gamma-1}dx = 1-\left((1-K)\frac{e_A^*}{e_A}\right)^{\gamma}.$$

The first result corresponds to the first case in the lemma. Adding the last two expressions and simplifying, we get the second case

$$\begin{aligned}
&\gamma \int_0^{(1-K)\frac{e_A^*}{e_A}} x^{\gamma-1}\left(x\frac{e_A}{e_A^*}+K\right)dx + \gamma\int_{(1-K)\frac{e_A^*}{e_A}}^1 x^{\gamma-1}dx\\
&= \frac{\gamma}{1+\gamma}\left((1-K)\frac{e_A^*}{e_A}\right)^{\gamma+1}\frac{e_A}{e_A^*}+\left((1-K)\frac{e_A^*}{e_A}\right)^{\gamma}K+1-\left((1-K)\frac{e_A^*}{e_A}\right)^{\gamma}\\
&= 1+\frac{\gamma}{1+\gamma}(1-K)^{\gamma+1}\left(\frac{e_A^*}{e_A}\right)^{\gamma}-(1-K)^{\gamma}\left(\frac{e_A^*}{e_A}\right)^{\gamma}(1-K)\\
&= 1-\frac{(1-K)^{1+\gamma}}{1+\gamma}\left(\frac{e_A^*}{e_A}\right)^{\gamma}.
\end{aligned} \tag{A19}$$

(b) and (c): The first and second derivatives can be computed straightforwardly from (a). □

**Lemma 5.** *For any* $e_B, e_B^* > 0$ *and* $K \in (0,1)$*, we have*

$$(a) \qquad \int_{-\infty}^{\infty} F\left(x\frac{e_B}{e_B^*}-K\right)f(x)dx = \begin{cases} 0 & \frac{e_B}{e_B^*}\le K\\ \frac{\left(e_B-Ke_B^*\right)^2}{2e_B^*e_B} & K<\frac{e_B}{e_B^*}\le 1+K\\ 1-(1+2K)\frac{e_B^*}{2e_B} & \frac{e_B}{e_B^*}>1+K,\end{cases} \tag{A20}$$

$$(b) \qquad \frac{\partial}{\partial e_B}\left(\int_{-\infty}^{\infty} F\left(x\frac{e_B}{e_B^*}-K\right)f(x)dx\right) = \begin{cases} 0 & \frac{e_B}{e_B^*}\le K\\ \frac{1}{2e_B^*}-\frac{e_B^*K^2}{2e_B^2} & K<\frac{e_B}{e_B^*}\le 1+K\\ (1+2K)\frac{e_B^*}{2e_B^2} & \frac{e_B}{e_B^*}>1+K,\end{cases} \tag{A21}$$

$$(c) \qquad \frac{\partial^2}{(\partial e_B)^2}\left(\int_{-\infty}^{\infty} F\left(x\frac{e_B}{e_B^*}-K\right)f(x)dx\right) = \begin{cases} 0 & \frac{e_B}{e_B^*}\le K\\ \frac{e_B^*K^2}{e_B^3} & K<\frac{e_B}{e_B^*}\le 1+K\\ -(1+2K)\frac{e_B^*}{e_B^3} & \frac{e_B}{e_B^*}>1+K.\end{cases} \tag{A22}$$

*Proof of Lemma 5.*
(a) We want to determine

$$\int_{-\infty}^{\infty} F\left(x\frac{e_B}{e_B^*}-K\right) f(x)\,dx = \int_0^1 F\left(x\frac{e_B}{e_B^*}-K\right) dx.$$

We distinguish three cases: i) $\frac{e_B}{e_B^*} \leq K$, ii) $K < \frac{e_B}{e_B^*} \leq 1+K$, iii) $\frac{e_B}{e_B^*} > 1+K$. In case i), the argument of $F$ is nonpositive, and we have

$$\int_0^1 F\left(x\frac{e_B}{e_B^*}-K\right) dx = \int_0^1 0dx = 0.$$

In case ii), we have

$$\begin{aligned}
\int_0^1 F\left(x\frac{e_B}{e_B^*}-K\right) dx &= \int_0^{K\frac{e_B^*}{e_B}} 0dx + \int_{K\frac{e_B^*}{e_B}}^1 \left(x\frac{e_B}{e_B^*}-K\right) dx \\
&= \left(\frac{e_B}{2e_B^*}x^2 - Kx\right)_{K\frac{e_B^*}{e_B}}^1 \\
&= \frac{e_B}{2e_B^*} - K - \frac{e_B}{2e_B^*}\left(K\frac{e_B^*}{e_B}\right)^2 + K^2\frac{e_B^*}{e_B} \\
&= \frac{\left(e_B - Ke_B^*\right)^2}{2e_B^* e_B}.
\end{aligned}$$

In case iii), we have

$$\begin{aligned}
\int_0^1 F\left(x\frac{e_B}{e_B^*}-K\right) dx &= \int_0^{K\frac{e_B^*}{e_B}} 0dx + \int_{K\frac{e_B^*}{e_B}}^{(1+K)\frac{e_B^*}{e_B}} \left(x\frac{e_B}{e_B^*}-K\right) dx + \int_{(1+K)\frac{e_B^*}{e_B}}^1 1dx \\
&= \left(\frac{e_B}{2e_B^*}x^2 - Kx\right)_{K\frac{e_B^*}{e_B}}^{(1+K)\frac{e_B^*}{e_B}} + 1 - (1+K)\frac{e_B^*}{e_B} \\
&= \left(1+2K+K^2\right)\frac{e_B^*}{2e_B} - \frac{K^2}{2}\frac{e_B^*}{e_B} + 1 - (1+2K)\frac{e_B^*}{e_B} \\
&= 1 - (1+2K)\frac{e_B^*}{2e_B} \\
&= \frac{2e_B - (1+2K)\,e_B^*}{2e_B}.
\end{aligned}$$

(b) and (c): The first and second derivatives can be computed straightforwardly from (a).

□

## A.2 Proof of Proposition 1

The game is solved by backward induction. In $t = 2$, both workers $i \in \{A, B\}$ choose the minimum effort, $e_{\min}$, as there are no incentives to justify higher effort.
Ability realizations $\theta_i$ and period-1 efforts $e_i$ are not observable. However, firms and workers have beliefs about efforts, denoted by $\tilde{e}_i$. They are confirmed in equilibrium, $\tilde{e}_i = e_i^*$.
After period 1, the incumbent firm can observe worker $i$'s output, $y_{i1L}$. Recalling (1), observed output and effort beliefs allow the firm to deduce the ability realization, which we denote as $\tilde{\theta}_i$ and which in equilibrium is equal to the actual ability realization $\theta_i$. The deduced beliefs about ability are

$$\tilde{\theta}_A = \frac{y_{A1L} - c_L}{d_L \tilde{e}_A}, \quad \tilde{\theta}_B = \frac{y_{B1L} - c_L}{d_L \tilde{e}_B}. \tag{A23}$$

We state the promotion rule as a function of the deduced ability levels $\tilde{\theta}_i$ rather than the observed output levels. The equilibrium promotion decision must be profit-maximizing and is based on both workers' expected period-2 productivity $\tilde{\theta}_i + q\tilde{e}_i$. Denote the set of deduced abilities $\tilde{\theta}_A$ and $\tilde{\theta}_B$ for which worker $A$ will be promoted by $T_A$ and the set of deduced abilities where $B$ is promoted by $T_B$.[19] Furthermore, denote the external firms' beliefs regarding $T_A$ and $T_B$ by $\tilde{T}_A$ and $\tilde{T}_B$, respectively.
We now consider the wages offered by the external firms. The outside firms can only observe the incumbent firm's promotion decision. Wage offers are therefore based on this observation, and on beliefs regarding the incumbent's promotion rule and the period-1 efforts. We consider the wage offers made by a representative external firm. We assume the external firm offers worker $i$ a wage rate of $w_{i2}^P$ if worker $i$ has been promoted and $w_{i2}^{NP}$ otherwise. The "2" indicates period 2. Due to perfect (Bertrand) competition, the (highest bidding) external firms offer wages that are equal to their expected gross profit (recall (4)). As the firm-specific human capital $S$ is assumed to be sufficiently large, the incumbent firm matches the external firms' wage offers (with probability $1 - \tau$, i.e., unless the incumbent firm mistakenly fails to make a counteroffer).
We start by considering the case where worker $A$ is promoted by the incumbent firm. In this case, the external wage offers are (where the expected value is from the point of view of the outside firm):

$$w_{A2}^P = c_L + d_L e_{\min} \left( E[\Theta_A | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in \tilde{T}_A] + q\tilde{e}_A \right), \tag{A24}$$

$$w_{B2}^{NP} = c_L + d_L e_{\min} \left( E[\Theta_B | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in \tilde{T}_A] + q\tilde{e}_B \right). \tag{A25}$$

[19] Note that beliefs about efforts, $\tilde{e}_i$, will also affect expected period-2 output and therefore wage offers and the promotion rule, but as they are beliefs, they are not uncertain. They are included in the promotion rule described by $T_A$ and $T_B$.

If worker $B$ is promoted, the wage offers by the external firm are

$$w_{A2}^{NP} = c_L + d_L e_{\min} \left( E[\Theta_A | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in \tilde{T}_B] + q\tilde{e}_A \right), \tag{A26}$$

$$w_{B2}^{P} = c_L + d_L e_{\min} \left( E[\Theta_B | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in \tilde{T}_B] + q\tilde{e}_B \right). \tag{A27}$$

We now turn to the incumbent firm's promotion decision at the end of period 1. Recall the period-2 outputs in the two job levels, (3) and (2). If the firm promotes worker $A$ (and hence does not promote worker $B$), the incumbent's expected period-2 profit is

$$\begin{aligned} \pi^{(P,NP)} = (1-\tau)\Big( & c_H + (1+S) d_H e_{\min} (\tilde{\theta}_A + q\tilde{e}_A)) \\ & + (c_L + (1+S) d_L e_{\min} (\tilde{\theta}_B + q\tilde{e}_B)) - (w_{A2}^{P} + w_{B2}^{NP})\Big) \end{aligned} \tag{A28}$$

Similarly, if worker $B$ is promoted, the firm's expected period-2 profit is

$$\begin{aligned} \pi^{(NP,P)} = (1-\tau)\Big( & (c_H + (1+S) d_H e_{\min} (\tilde{\theta}_B + q\tilde{e}_B)) \\ & + (c_L + (1+S) d_L e_{\min} (\tilde{\theta}_A + q\tilde{e}_A)) - (w_{A2}^{NP} + w_{B2}^{P})\Big). \end{aligned} \tag{A29}$$

It follows that the firm promotes worker $A$ if and only if

$$\pi^{(P,NP)} > \pi^{(NP,P)} \iff (1+S)(d_H - d_L) e_{\min} (\tilde{\theta}_A + q\tilde{e}_A - (\tilde{\theta}_B + q\tilde{e}_B)) > w_{A2}^{P} + w_{B2}^{NP} - w_{A2}^{NP} - w_{B2}^{P} \tag{A30}$$

Recalling that job $H$ is more responsive to period-2 productivity $\theta_i + qe_i$ than job $L$ ($d_H > d_L$), the obvious candidate equilibrium promotion rule is that worker $A$ is promoted if and only if $\tilde{\theta}_A + q\tilde{e}_A > \tilde{\theta}_B + q\tilde{e}_B$. In order to prove that this is an equilibrium promotion rule, we focus attention on the RHS of (A30).

Suppose, in equilibrium, worker $A$ is indeed promoted iff $\tilde{\theta}_A + q\tilde{e}_A > \tilde{\theta}_B + q\tilde{e}_B$. In equilibrium, outside firms correctly anticipate the promotion rule. Therefore, $\tilde{T}_A = T_A$ and $\tilde{T}_B = T_B$. Recalling the wage offers (A24)–(A27), the RHS of (A30) is then equal to

$$\begin{aligned} d_L e_{\min} \Big( & E[\Theta_A | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in T_A] - E[\Theta_A | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in T_B] \\ & + E[\Theta_B | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in T_A] - E[\Theta_B | (\tilde{\Theta}_A, \tilde{\Theta}_B) \in T_B] \Big). \end{aligned} \tag{A31}$$

We write our random variables as $\Theta_i = \mu + \varepsilon_i$ and note that $\mu = \frac{1}{2}$ while $\varepsilon_A$ and $\varepsilon_B$ are random variables with mean zero that are identically, independently, and symmetrically distributed on $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Using this definition and the candidate promotion rule, the preceding expression can be written as

$$d_L e_{\min}\left(\overbrace{E[\varepsilon_A|\tilde{\varepsilon}_A+q\tilde{e}_A>\tilde{\varepsilon}_B+q\tilde{e}_B]-E[\varepsilon_A|\tilde{\varepsilon}_A+q\tilde{e}_A<\tilde{\varepsilon}_B+q\tilde{e}_B]}^{=:\alpha}\right.$$
$$\left.+\underbrace{E[\varepsilon_B|\tilde{\varepsilon}_A+q\tilde{e}_A>\tilde{\varepsilon}_B+q\tilde{e}_B]-E[\varepsilon_B|\tilde{\varepsilon}_A+q\tilde{e}_A<\tilde{\varepsilon}_B+q\tilde{e}_B]}_{=:\beta}\right). \tag{A32}$$

In the following, we show that $\beta=-\alpha$. Consider expression $\beta$. As $\varepsilon_A$ and $\varepsilon_B$ are identically, independently, and symmetrically distributed with mean zero, the variables $\varepsilon_A$ and $-\varepsilon_B$ as well as $-\varepsilon_A$ and $\varepsilon_B$ are i.i.d. Therefore, we can replace $\varepsilon_A$ by $-\varepsilon_B$ (and vice versa) everywhere in $\beta$. Moreover, in equilibrium, beliefs about efforts $\tilde{e}_A$ and $\tilde{e}_B$ are correct, i.e., the distributions of $\tilde{\varepsilon}_A$ (resp. $\tilde{\varepsilon}_B$) and $\varepsilon_A$ (resp. $\varepsilon_B$) are the same. We can therefore also replace $\tilde{\varepsilon}_A$ by $-\tilde{\varepsilon}_B$ and vice versa. We obtain

$$\begin{aligned}\beta&=E[-\varepsilon_A|-\tilde{\varepsilon}_B+q\tilde{e}_A>-\tilde{\varepsilon}_A+q\tilde{e}_B]-E[-\varepsilon_A|-\tilde{\varepsilon}_B+q\tilde{e}_A<-\tilde{\varepsilon}_A+q\tilde{e}_B]\\&=-\left(E[\varepsilon_A|-\tilde{\varepsilon}_B+q\tilde{e}_A>-\tilde{\varepsilon}_A+q\tilde{e}_B]-E[\varepsilon_A|-\tilde{\varepsilon}_B+q\tilde{e}_A<-\tilde{\varepsilon}_A+q\tilde{e}_B]\right)\\&=-\left(E[\varepsilon_A|\tilde{\varepsilon}_A+q\tilde{e}_A>\tilde{\varepsilon}_B+q\tilde{e}_B]-E[\varepsilon_A|\tilde{\varepsilon}_A+q\tilde{e}_A<\tilde{\varepsilon}_B+q\tilde{e}_B]\right)\\&=-\alpha.\end{aligned} \tag{A33}$$

It follows that the RHS of (A30) is equal to zero, $w_{A2}^{P}+w_{B2}^{NP}-w_{A2}^{NP}-w_{B2}^{P}=0$, which can equivalently be expressed as

$$w_{A2}^{P}-w_{A2}^{NP}=w_{B2}^{P}-w_{B2}^{NP}, \tag{A34}$$

which means that the absolute (period 2) wage premium of getting promoted is the same for both workers. This property is a result of the symmetry of the ability distributions around their means.

With the RHS of (A30) being equal to zero, we see that $\pi^{(P,NP)}>\pi^{(NP,P)}$ is equivalent to $\tilde{\theta}_A+q\tilde{e}_A>\tilde{\theta}_B+q\tilde{e}_B$, the candidate promotion rule. Therefore, this promotion rule is profit-maximizing and part of an equilibrium, i.e., the incumbent firm does not have an incentive to deviate from it.

The next step is to determine the two workers' period-1 effort choices. We start by considering worker $A$. In equilibrium, worker $A$ anticipates to be promoted if and only if (below we replace output $y_{A1L}$ by actual output which is a function of actual effort $e_A$ that is chosen by the workers,

$$
\begin{aligned}
&\tilde{\theta}_A + q\tilde{e}_A > \tilde{\theta}_B + q\tilde{e}_B \\
\Longleftrightarrow\ & \frac{y_{A1L} - c_L}{d_L \tilde{e}_A} + q\tilde{e}_A > \frac{y_{B1L} - c_L}{d_L \tilde{e}_B} + q\tilde{e}_B \\
\Longleftrightarrow\ & \frac{(c_L + d_L e_A \theta_A) - c_L}{d_L \tilde{e}_A} + q\tilde{e}_A > \frac{(c_L + d_L e_B \theta_B) - c_L}{d_L \tilde{e}_B} + q\tilde{e}_B \\
\Longleftrightarrow\ & \frac{e_A \theta_A}{\tilde{e}_A} + q\tilde{e}_A > \frac{e_B \theta_B}{\tilde{e}_B} + q\tilde{e}_B \\
\Longleftrightarrow\ & \theta_B < \theta_A \frac{e_A \tilde{e}_B}{e_B \tilde{e}_A} + \frac{q(\tilde{e}_A - \tilde{e}_B)\tilde{e}_B}{e_B}.
\end{aligned}
\tag{A35}
$$

Worker $A$'s *subjective* promotion probability (using pdf $\hat{f}$) can now be stated as

$$
\hat{P}_A = \int_{-\infty}^{\infty} F\left(x \frac{e_A \tilde{e}_B}{e_B \tilde{e}_A} + \frac{q(\tilde{e}_A - \tilde{e}_B)\tilde{e}_B}{e_B}\right) \hat{f}\,(x)\, dx. \tag{A36}
$$

We continue the analysis supposing that efforts and beliefs imply that $\hat{P}_A \in (0,1)$, in line with our assumption that none of the workers is promoted with certainty.
Differentiating with respect to $A$'s choice variable $e_A$, we obtain

$$
\frac{\partial \hat{P}_A}{\partial e_A} = \int_{-\infty}^{\infty} f\left(x \frac{e_A \tilde{e}_B}{e_B \tilde{e}_A} + \frac{q(\tilde{e}_A - \tilde{e}_B)\tilde{e}_B}{e_B}\right) \left(x \frac{\tilde{e}_B}{e_B \tilde{e}_A}\right) \hat{f}\,(x)\, dx. \tag{A37}
$$

Denote equilibrium efforts by $e_A^*$ and $e_B^*$. As beliefs regarding efforts are confirmed in equilibrium, $e_A = \tilde{e}_A = e_A^*$ and $e_B = \tilde{e}_B = e_B^*$, the latter expression simplifies to

$$
\left.\frac{\partial \hat{P}_A}{\partial e_A}\right|_{(e_A^*, e_B^*)} = \int_{-\infty}^{\infty} f\left(x + q(e_A^* - e_B^*)\right) \frac{x}{e_A^*} \hat{f}\,(x)\, dx. \tag{A38}
$$

We now turn to $A$'s problem of maximizing expected payoff, which, in general terms, can be expressed as

$$
\hat{P}_A \times (\text{expected payoff given } P) + (1 - \hat{P}_A) \times (\text{expected payoff given } NP). \tag{A39}
$$

This equals

$$
\begin{aligned}
&\hat{P}_A w_{A2}^P + \left(1-\hat{P}_A\right) w_{A2}^{NP} - c(e_A) \\
&= \hat{P}_A \left(w_{A2}^P - w_{A2}^{NP}\right) + w_{A2}^{NP} - c(e_A) \\
&= \hat{P}_A \Big( \Big[c_L + d_L e_{\min} \Big( E[\Theta_A | \tilde{\Theta}_A + q\tilde{e}_A > \tilde{\Theta}_B + q\tilde{e}_B] + q\tilde{e}_A \Big)\Big] \\
&\qquad - \Big[c_L + d_L e_{\min} \Big( E[\Theta_A | \tilde{\Theta}_A + q\tilde{e}_A < \tilde{\Theta}_B + q\tilde{e}_B] + q\tilde{e}_A \Big)\Big] \Big) \\
&\qquad + \Big[c_L + d_L e_{\min} \Big( E[\Theta_A | \tilde{\Theta}_A + q\tilde{e}_A < \tilde{\Theta}_B + q\tilde{e}_B] + q\tilde{e}_A \Big)\Big] - c(e_A) \\
&= \hat{P}_A d_L e_{\min} \Big( E[\Theta_A | \tilde{\Theta}_A + q\tilde{e}_A > \tilde{\Theta}_B + q\tilde{e}_B] - E[\Theta_A | \tilde{\Theta}_A + q\tilde{e}_A < \tilde{\Theta}_B + q\tilde{e}_B] \Big) \\
&\qquad + c_L + d_L e_{\min} \Big( E[\Theta_A | \tilde{\Theta}_A + q\tilde{e}_A < \tilde{\Theta}_B + q\tilde{e}_B] + q\tilde{e}_A \Big) - c(e_A).
\end{aligned}
\tag{A40}
$$

Note that $A$'s choice variable $e_A$ appears only in the cost function and in the probability of winning $\hat{P}_A$, see (A36). The reason is that wages only depend on beliefs regarding effort (not the actual effort choices). The overconfident worker $A$ is aware of how the firms form expectations about $A$'s ability (agree to disagree) and takes this into account in (A40) above. In equilibrium, beliefs about the efforts of both workers are correct, $\tilde{e}_i = e_i^*$, $i \in \{A, B\}$. As a consequence, beliefs about ability realizations are correct, $\tilde{\theta}_i = \theta_i$, which implies $\tilde{\Theta}_i = \Theta_i$. Thus, the first-order condition to worker $A$'s decision problem, evaluated in equilibrium, is

$$
\begin{aligned}
c'(e_A^*) = d_L e_{\min} \left.\frac{\partial \hat{P}_A}{\partial e_A}\right|_{(e_A^*, e_B^*)} &\Big( E[\Theta_A | \Theta_A + q e_A^* > \Theta_B + q e_B^*] \\
&- E[\Theta_A | \Theta_A + q e_A^* < \Theta_B + q e_B^*] \Big).
\end{aligned}
\tag{A41}
$$

By symmetry, we have for worker $B$, where the pdf $f$ replaces $\hat{f}$,

$$
P_B = \int_{-\infty}^{\infty} F\left( x \frac{e_B \tilde{e}_A}{e_A \tilde{e}_B} + \frac{q(\tilde{e}_B - \tilde{e}_A)\tilde{e}_A}{e_A} \right) f(x)\, dx, \tag{A42}
$$

$$
\left.\frac{\partial P_B}{\partial e_B}\right|_{(e_A^*, e_B^*)} = \int_{-\infty}^{\infty} f\left(x - q(e_A^* - e_B^*)\right) \frac{x}{e_B^*} f(x)\, dx. \tag{A43}
$$

Using similar steps as above, $B$'s first-order condition (evaluated in equilibrium) can be derived as follows (exploiting (A34), expressing the difference in expected values in terms of $\Theta_A$ rather than $\Theta_B$):

$$
\begin{aligned}
c'(e_B^*) = d_L e_{\min} \left.\frac{\partial P_B}{\partial e_B}\right|_{(e_A^*, e_B^*)} &\Big( E[\Theta_A | \Theta_A + q e_A^* > \Theta_B + q e_B^*] \\
&- E[\Theta_A | \Theta_A + q e_A^* < \Theta_B + q e_B^*] \Big).
\end{aligned}
\tag{A44}
$$

In order to simplify notation, define $K := q(e_A^* - e_B^*)$.[20] The above first-order conditions can be written as

$$c'(e_A^*)e_A^* = d_L e_{\min} \int_{-\infty}^{\infty} f(x+K)\, x \hat{f}(x)\, dx \\ \Big(E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B]\Big), \quad \text{(A45)}$$

$$c'(e_B^*)e_B^* = d_L e_{\min} \int_{-\infty}^{\infty} f(x-K)\, x f(x)\, dx \\ \Big(E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B]\Big). \quad \text{(A46)}$$

In the following, we prove that $e_A^* > e_B^*$. Recall that, by assumption, $K \in (-1, 1)$. By contradiction, assume that $e_A^* \leq e_B^*$, which is equivalent to $K \in (-1, 0]$. For this case, the two integrals above are given in Lemma 3. We now demonstrate that, for $K \in (-1, 0]$,

$$\int_{-\infty}^{\infty} f(x+K)\, x \hat{f}(x)\, dx > \int_{-\infty}^{\infty} f(x-K)\, x f(x)\, dx \\ \iff \frac{\gamma}{1+\gamma}(1 + K(-K)^{\gamma}) > \frac{1}{2}(1+K)^2 \quad \text{(A47)}$$

is true. First consider the case $K = 0$. Then the inequality simplifies to $\frac{\gamma}{1+\gamma} > \frac{1}{2} \Leftrightarrow 2\gamma > 1 + \gamma \Leftrightarrow \gamma > 1$, which is always fulfilled. Now consider the case $K \in (-1, 0)$. The above observation that $\frac{\gamma}{1+\gamma} > \frac{1}{2}$ implies that we only need to show that

$$\begin{aligned} & 1 + K(-K)^{\gamma} > (1+K)^2 \\ \Leftrightarrow \quad & 1 + K(-K)^{\gamma} > 1 + 2K + K^2 \\ \Leftrightarrow \quad & K(-K)^{\gamma} > 2K + K^2 \\ \Leftrightarrow \quad & (-K)^{\gamma} < 2 + K. \end{aligned}$$

Notice that $(-K)^{\gamma} < 1$, while $2 + K > 1$, so inequality (A47) is fulfilled for all $K \in (-1, 0]$. Thus, the RHS of (A45) would be larger than the RHS of (A46), which implies that the LHS of (A45) is larger than the LHS of (A46). As $c'(x)x$ is increasing in $x$, the latter contradicts $e_A^* \leq e_B^*$. Therefore, $e_A^* \leq e_B^*$ is not a solution to the pair of first-order conditions. Therefore, if there is an equilibrium that is characterized by the first-order conditions, it must satisfy $e_A^* > e_B^*$.

Having established that $e_A^* > e_B^*$, implying that $K \in (0, 1)$, in the following, we rewrite these conditions, inserting our distributional assumptions.
First, applying Lemma 3 for $K \in (0, 1)$, we can write the equilibrium marginal promotion

[20] In the paper, we assume that, given all other model parameters, the cost function is sufficiently convex such that the equilibrium difference in transferable human capital between the workers is less than one, $|q(e_A^* - e_B^*)| < 1$. This assumption ensures that both workers are promoted with a positive probability.

probabilities as

$$\left.\frac{\partial \hat{P}_A}{\partial e_A}\right|_{(e_A^*,e_B^*)} = \frac{1}{e_A^*}\int_{-\infty}^{\infty} f\left(x+K\right)x\hat{f}\left(x\right)dx = \frac{1}{e_A^*}\frac{\gamma(1-K)^{\gamma+1}}{\gamma+1},$$
$$\left.\frac{\partial P_B}{\partial e_B}\right|_{(e_A^*,e_B^*)} = \frac{1}{e_B^*}\int_{-\infty}^{\infty} f\left(x-K\right)xf\left(x\right)dx = \frac{1}{e_B^*}\frac{1-K^2}{2}. \tag{A48}$$

Second, Lemma 2 part (c) provides the difference of conditional expectations,

$$\Big(E[\Theta_A|\Theta_A+K>\Theta_B]-E[\Theta_A|\Theta_A+K<\Theta_B]\Big) = \frac{1+2K}{3(1+2K-K^2)}. \tag{A49}$$

This allows us to write (A41) and (A44) as

$$c'(e_A^*)e_A^* = d_L e_{\min}\frac{\gamma(1-K)^{\gamma+1}}{3(\gamma+1)}\frac{1+2K}{1+2K-K^2},$$
$$c'(e_B^*)e_B^* = d_L e_{\min}\frac{1}{6}(1-K^2)\frac{1+2K}{1+2K-K^2}. \tag{A50}$$

Recall that $K$ is a function of the equilibrium efforts, which means that we can only implicitly characterize equilibrium efforts.

### A.2.1 Second order conditions

We continue with deriving sufficient second-order conditions such that (A50) indeed characterizes an equilibrium. For this, we look at each worker's expected deviation payoff, i.e., worker $i$'s payoff as a function of $e_i$ given that the other worker, $j$, plays the above Nash equilibrium candidate effort $e_j^*$, and given that all beliefs are also equal to the above two candidate efforts $(e_A^*, e_B^*)$.

Start with worker $A$'s problem. The overconfident $A$'s subjective probability of winning as a function of $e_A$, evaluated at firms' candidate equilibrium beliefs and worker $B$'s candidate equilibrium effort, $\tilde{e}_A = e_A^*, \tilde{e}_B = e_B = e_B^*$, is

$$\left.\hat{P}_A\right|_{\tilde{e}_A=e_A^*,\tilde{e}_B=e_B=e_B^*} = \left.\int_{-\infty}^{\infty} F\left(x\frac{e_A\tilde{e}_B}{e_B\tilde{e}_A}+\frac{q(\tilde{e}_A-\tilde{e}_B)\tilde{e}_B}{e_B}\right)\hat{f}\left(x\right)dx\right|_{\tilde{e}_A=e_A^*,\tilde{e}_B=e_B=e_B^*}$$
$$= \int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*}+q(e_A^*-e_B^*)\right)\hat{f}\left(x\right)dx. \tag{A51}$$

Recall $A$'s expected payoff in the last line of (A40), which we also evaluate at $\tilde{e}_A = e_A^*$,

$\tilde{e}_B = e_B = e_B^*$, $\tilde{\theta}_i = \theta_i$, again using $K := q(e_A^* - e_B^*)$ to get

$$\int_{-\infty}^{\infty} F\left(x\frac{e_A}{e_A^*} + K\right) \hat{f}(x)\, dx\; d_L e_{\min}\Big(E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B]\Big)$$
$$+ c_L + d_L e_{\min}\left(E[\Theta_A|\Theta_A + K < \Theta_B] + qe_A^*\right) - c(e_A), \tag{A52}$$

where only the integral and the cost term depend on $A$'s choice variable $e_A$. The integral is multiplied by a positive constant, recall (A49). Lemma 4 derives the first and second derivatives of the above integral. The integral is (once) continuously differentiable. As the second derivative of the integral is either zero or negative, while the cost function is convex, we conclude that $e_A = e_A^*$ is a best response for worker $A$.

We repeat similar steps for worker $B$. Start with the winning probability:

$$\begin{aligned} P_B|_{e_A=\tilde{e}_A=e_A^*, \tilde{e}_B=e_B^*} &= \int_{-\infty}^{\infty} F\left(x\frac{e_B\tilde{e}_A}{e_A\tilde{e}_B} + \frac{q(\tilde{e}_B - \tilde{e}_A)\tilde{e}_A}{e_A}\right) f(x)\, dx\Bigg|_{e_A=\tilde{e}_A=e_A^*, \tilde{e}_B=e_B^*} \\ &= \int_{-\infty}^{\infty} F\left(x\frac{e_B}{e_B^*} - q(e_A^* - e_B^*)\right) f(x)\, dx. \end{aligned} \tag{A53}$$

The payoff of worker $B$ evaluated at the equilibrium candidate and beliefs is:

$$\int_{-\infty}^{\infty} F\left(x\frac{e_B}{e_B^*} - K\right) f(x)\, dx\; d_L e_{\min}\Big(E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B]\Big)$$
$$+ c_L + d_L e_{\min}\left(E[\Theta_B|\Theta_A + K > \Theta_B] + qe_B^*\right) - c(e_B). \tag{A54}$$

Similar to worker $A$'s problem, only the integral and the cost term depend on the choice variable $e_B$. The integral is multiplied by a positive constant, recall (A49). Lemma 5 derives the first and second derivatives of the above integral. The integral is (once) continuously differentiable. Note that, in contrast to worker $A$'s problem, the second derivative of the integral can be positive, while the cost function is convex. We conclude that $e_B = e_B^*$ is a best response for worker $B$ only if suitable parameters are identified. In the numerical examples that we provide, the second-order conditions are satisfied.

### A.3 Proof of Corollary 1

(a) **Worker $A$ is promoted with a higher probability.**

According to the equilibrium promotion rule, $A$ is promoted if and only if (using the notation $K := q(e_A^* - e_B^*)$)

$$\theta_A + qe_A^* > \theta_B + qe_B^* \iff \theta_B < \theta_A + K. \tag{A55}$$

The probability of that event is denoted by $P(\Theta_A + K > \Theta_B)$ and is given by (A4). The probability that worker $B$ is promoted is denoted by $P(\Theta_A + K < \Theta_B)$ and is given by (A5). We have that

$$\begin{aligned} P(\Theta_A + K > \Theta_B) &= \int_{-\infty}^{\infty} F(x+K) f(x) dx \\ &> \int_{-\infty}^{\infty} F(x) f(x) dx = \frac{1}{2}, \end{aligned} \tag{A56}$$

since $e_A^* > e_B^*$ and $K > 0$. Thus, the probability of $A$ being promoted is larger than $\frac{1}{2}$, implying that worker $B$'s promotion probability is less than $\frac{1}{2}$.

(b) **Worker $A$ receives a higher period-2 wage than worker $B$**

Recall (A24)–(A27), and insert the equilibrium promotion rule $\theta_A + qe_A^* > \theta_B + qe_B^*$. The two workers' period-2 wages for both feasible promotion events are

$$w_{A2}^{NP} = c_L + d_L e_{\min} \left( E\left[\Theta_A \middle| \Theta_A + qe_A^* < \Theta_B + qe_B^*\right] + qe_A^* \right), \tag{A57}$$

$$w_{A2}^{P} = c_L + d_L e_{\min} \left( E\left[\Theta_A \middle| \Theta_A + qe_A^* > \Theta_B + qe_B^*\right] + qe_A^* \right), \tag{A58}$$

$$w_{B2}^{NP} = c_L + d_L e_{\min} \left( E\left[\Theta_B \middle| \Theta_B + qe_B^* < \Theta_A + qe_A^*\right] + qe_B^* \right), \tag{A59}$$

$$w_{B2}^{P} = c_L + d_L e_{\min} \left( E\left[\Theta_B \middle| \Theta_B + qe_B^* > \Theta_A + qe_A^*\right] + qe_B^* \right). \tag{A60}$$

We need to show that both $w_{A2}^{NP} > w_{B2}^{NP}$ and $w_{A2}^{P} > w_{B2}^{P}$. Note that $w_{A2}^{NP} > w_{B2}^{NP}$ is equivalent to

$$q(e_A^* - e_B^*) + E\left[\Theta_A \middle| \Theta_A + qe_A^* < \Theta_B + qe_B^*\right] > E\left[\Theta_B \middle| \Theta_B + qe_B^* < \Theta_A + qe_A^*\right]$$

Denoting $K := q(e_A^* - e_B^*) \in (0,1)$, and inserting two conditional expectations computed in Lemma 2, cases (b) and (d), this simplifies to

$$\begin{aligned} & K + E[\Theta_A | \Theta_A + K < \Theta_B] > E[\Theta_B | \Theta_A + K > \Theta_B] \\ \iff \quad & K + \frac{1-K}{3} > \frac{1 + 3K - K^3}{3(1 + 2K - K^2)} \\ \iff \quad & 1 + 2K > \frac{1 + 3K - K^3}{1 + 2K - K^2} \\ \iff \quad & K + 3K^2 - K^3 > 0. \end{aligned} \tag{A61}$$

Since $K \in (0,1)$ and thus $K > K^3$, this always holds. Thus, $w_{A2}^{NP} > w_{B2}^{NP}$. By (A34), this implies $w_{A2}^{P} > w_{B2}^{P}$.

(c) **Worker $A$ receives a higher expected period-2 wage .**

The expected period-2 wages for both workers are

$$P_A(w_{A2}^P - w_{A2}^{NP}) + w_{A2}^{NP} \tag{A62}$$

$$P_B(w_{B2}^P - w_{B2}^{NP}) + w_{B2}^{NP} \tag{A63}$$

The proof follows directly from parts (b) and (c), i.e., a larger promotion probability, $P_A > P_B$, combined with $A$ receiving a larger wage than $B$ if not promoted, $w_{A2}^{NP} > w_{B2}^{NP}$. By (A34), the differences in parentheses are equal.

(d) **Upon promotion, in expectation, worker $A$'s ability is smaller than worker $B$'s**

Recall that in the event of promotion, worker $A$'s ability $\theta_A$ satisfies

$$\begin{aligned} &\theta_A + qe_A^* > \theta_B + qe_B^* \\ \iff &\theta_A > \theta_B - q(e_A^* - e_B^*). \end{aligned} \tag{A64}$$

The expected value of $\Theta_A$ in this event is found in part (a) of Lemma 2, for $K = q(e_A^* - e_B^*) \in (0,1)$. For worker $B$'s promotion event,

$$\begin{aligned} &\theta_A + qe_A^* < \theta_B + qe_B^* \\ \iff &\theta_B > \theta_A + q(e_A^* - e_B^*), \end{aligned} \tag{A65}$$

the relevant expected value is found in part (e) of Lemma 2, again for $K = q(e_A^* - e_B^*) \in (0,1)$. The expected values in part (a) and (e) can be written as

$$\begin{aligned} (a) \quad & E[\Theta_A | \Theta_A > \Theta_B - K] \\ (e) \quad & E[\Theta_B | \Theta_B > \Theta_A + K]. \end{aligned} \tag{A66}$$

Obviously, $E[\Theta_A|\Theta_A > \Theta_B - K] < E[\Theta_B|\Theta_B > \Theta_A + K]$, as $\Theta_A$ and $\Theta_B$ are i.i.d. and $K > 0$. This can of course also be confirmed by comparing the respective expressions.

(e) **Worker $A$ has larger transferable human capital**

This follows directly from $e_A^* > e_B^*$, which implies larger human capital for $A$, $qe_A^* > qe_B^*$.

## A.4 Period-1 Wages

In this section we analyze period-1 wage payments $w^*_{A1}$ and $w^*_{B1}$. In period 1, the incumbent expects a net profit from worker $A$ equal to

$$\begin{aligned} & c_L + d_L E\left[\Theta_A\right] e^*_A - w^*_{A1} \\ & = c_L + \frac{d_L}{2} e^*_A - w^*_{A1}, \end{aligned} \tag{A67}$$

and from worker $B$ equal to

$$\begin{aligned} & c_L + d_L E\left[\Theta_B\right] e^*_B - w^*_{B1} \\ & = c_L + \frac{d_L}{2} e^*_B - w^*_{B1}. \end{aligned} \tag{A68}$$

We assume that, due to a competitive labor market, the incumbent expects zero total net profit from both periods. Thus, in order to determine $w^*_{A1}$ and $w^*_{B1}$ we need to derive the total expected net profit from each worker over both periods.
We continue with deriving the incumbent's expected output in period 2 (recalling that the workers stay at the incumbent firm with probability $1-\tau$). The expected output from worker $A$ is (recalling (A28) and (A29))

$$\begin{aligned} & (1-\tau)\Big(P\left(\Theta_A + qe^*_A > \Theta_B + qe^*_B\right)\left(c_H + (1+S)\, d_H e_{\min} E\left[\Theta_A + qe^*_A \middle|\, \Theta_A + qe^*_A > \Theta_B + qe^*_B\right]\right) \\ & + P\left(\Theta_A + qe^*_A < \Theta_B + qe^*_B\right)\left(c_L + (1+S)\, d_L e_{\min} E\left[\Theta_A + qe^*_A \middle|\, \Theta_A + qe^*_A < \Theta_B + qe^*_B\right]\right)\Big). \end{aligned} \tag{A69}$$

Similarly, for worker $B$ we have

$$\begin{aligned} & (1-\tau)\Big(P\left(\Theta_A + qe^*_A > \Theta_B + qe^*_B\right)\left(c_L + (1+S)\, d_L e_{\min} E\left[\Theta_B + qe^*_B \middle|\, \Theta_A + qe^*_A > \Theta_B + qe^*_B\right]\right) \\ & + P\left(\Theta_A + qe^*_A < \Theta_B + qe^*_B\right)\left(c_H + (1+S)\, d_H e_{\min} E\left[\Theta_B + qe^*_B \middle|\, \Theta_A + qe^*_A < \Theta_B + qe^*_B\right]\right)\Big). \end{aligned} \tag{A70}$$

As a final ingredient of the incumbent's total profit, consider the expected wage payment to each worker in the second period, recalling (A57)–(A60). For worker $A$ this is

$$\begin{aligned} & (1-\tau)\Big(c_L + d_L e_{\min}\Big(P\left(\Theta_A + qe^*_A > \Theta_B + qe^*_B\right)\left(E\left[\Theta_A \middle|\, \Theta_A + qe^*_A > \Theta_B + qe^*_B\right] + qe^*_A\right) \\ & + P\left(\Theta_A + qe^*_A < \Theta_B + qe^*_B\right)\left(E\left[\Theta_A \middle|\, \Theta_A + qe^*_A < \Theta_B + qe^*_B\right] + qe^*_A\right)\Big)\Big), \end{aligned} \tag{A71}$$

where

$$P\left(\Theta_A + qe_A^* > \Theta_B + qe_B^*\right) E\left[\Theta_A \middle| \Theta_A + qe_A^* > \Theta_B + qe_B^*\right] \\ + P\left(\Theta_A + qe_A^* < \Theta_B + qe_B^*\right) E\left[\Theta_A \middle| \Theta_A + qe_A^* < \Theta_B + qe_B^*\right] = \frac{1}{2}. \tag{A72}$$

The latter follows from the law of total expectation and can easily be verified using the results in Lemmas 1 and 2. So the expected wage payment simplifies to

$$(1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_A^*\right)\right). \tag{A73}$$

Similarly, the incumbent's expected period-2 wage payment to worker $B$ is

$$(1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_B^*\right)\right). \tag{A74}$$

Combining the above results (A67), (A69), and (A73), the incumbent's total expected net profit (output minus wages) from worker $A$ is

$$c_L + \frac{d_L}{2} e_A^* - w_{A1}^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_A^*\right)\right) + (1-\tau) \cdot \\ \Big(P\left(\Theta_A + qe_A^* > \Theta_B + qe_B^*\right)\left(c_H + (1+S)\, d_H e_{\min} E\left[\Theta_A + qe_A^* \middle| \Theta_A + qe_A^* > \Theta_B + qe_B^*\right]\right) \\ + P\left(\Theta_A + qe_A^* < \Theta_B + qe_B^*\right)\left(c_L + (1+S)\, d_L e_{\min} E\left[\Theta_A + qe_A^* \middle| \Theta_A + qe_A^* < \Theta_B + qe_B^*\right]\right)\Big). \tag{A75}$$

If we impose a zero-profit condition on worker $A$, then the incumbent needs to pay worker $A$ a period-1 wage of

$$w_{A1}^* = c_L + \frac{d_L}{2} e_A^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_A^*\right)\right) + (1-\tau) \cdot \\ \Big(P\left(\Theta_A + qe_A^* > \Theta_B + qe_B^*\right)\left(c_H + (1+S)\, d_H e_{\min} E\left[\Theta_A + qe_A^* \middle| \Theta_A + qe_A^* > \Theta_B + qe_B^*\right]\right) \\ + P\left(\Theta_A + qe_A^* < \Theta_B + qe_B^*\right)\left(c_L + (1+S)\, d_L e_{\min} E\left[\Theta_A + qe_A^* \middle| \Theta_A + qe_A^* < \Theta_B + qe_B^*\right]\right)\Big). \tag{A76}$$

Denoting $K := q(e_A^* - q_B^*)$, we can write this as

$$w_{A1}^* = c_L + \frac{d_L}{2} e_A^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_A^*\right)\right) + (1-\tau) \cdot \\ \Big(P\left(\Theta_A + K > \Theta_B\right)\left(c_H + (1+S)\, d_H e_{\min}(E\left[\Theta_A \middle| \Theta_A + K > \Theta_B\right] + qe_A^*)\right) \\ + P\left(\Theta_A + K < \Theta_B\right)\left(c_L + (1+S)\, d_L e_{\min}(E\left[\Theta_A \middle| \Theta_A + K < \Theta_B\right] + qe_A^*)\right)\Big). \tag{A77}$$

Recall the promotion probabilities (A4) and (A5). Furthermore, insert conditional expectations developed in Lemma 2 to write the above as

$$\begin{aligned} w_{A1}^* = c_L + \frac{d_L}{2} e_A^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + q e_A^*\right)\right) + (1-\tau)\cdot \\ \left(\frac{1}{2}(1+2K-K^2)\left(c_H + (1+S)\, d_H e_{\min}\left(\frac{2+3K-3K^2+K^3}{3(1+2K-K^2)} + q e_A^*\right)\right)\right. \\ \left. + \frac{1}{2}(1-2K+K^2)\left(c_L + (1+S)\, d_L e_{\min}\left(\frac{1-K}{3} + q e_A^*\right)\right)\right). \end{aligned} \tag{A78}$$

Similarly, collecting (A68), (A70), and (A74), worker $B$'s period-1 wage would have to be

$$\begin{aligned} w_{B1}^* = c_L + \frac{d_L}{2} e_B^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + q e_B^*\right)\right) + (1-\tau)\cdot \\ \Big(P\left(\Theta_A + q e_A^* > \Theta_B + q e_B^*\right)\left(c_L + (1+S)\, d_L e_{\min} E\left[\Theta_B + q e_B^* \middle| \Theta_A + q e_A^* > \Theta_B + q e_B^*\right]\right) \\ + P\left(\Theta_A + q e_A^* < \Theta_B + q e_B^*\right)\left(c_H + (1+S)\, d_H e_{\min} E\left[\Theta_B + q e_B^* \middle| \Theta_A + q e_A^* < \Theta_B + q e_B^*\right]\right)\Big). \end{aligned} \tag{A79}$$

Denoting $K := q(e_A^* - q_B^*)$, we can write this as

$$\begin{aligned} w_{B1}^* = c_L + \frac{d_L}{2} e_B^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + q e_B^*\right)\right) + (1-\tau)\cdot \\ \Big(P\left(\Theta_A + K > \Theta_B\right)\left(c_L + (1+S)\, d_L e_{\min}\left(E\left[\Theta_B \middle| \Theta_A + K > \Theta_B\right] + q e_B^*\right)\right) \\ + P\left(\Theta_A + K < \Theta_B\right)\left(c_H + (1+S)\, d_H e_{\min}\left(E\left[\Theta_B \middle| \Theta_A + K < \Theta_B\right] + q e_B^*\right)\right)\Big). \end{aligned} \tag{A80}$$

Recall the promotion probabilities (A4) and (A5). Furthermore, insert conditional expectations developed in Lemma 2 to write the above as

$$\begin{aligned} w_{B1}^* = c_L + \frac{d_L}{2} e_B^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + q e_B^*\right)\right) + (1-\tau)\cdot \\ \left(\frac{1}{2}(1+2K-K^2)\left(c_L + (1+S)\, d_L e_{\min}\left(\frac{1+3K-K^3}{3(1+2K-K^2)} + q e_B^*\right)\right)\right. \\ \left. + \frac{1}{2}(1-2K+K^2)\left(c_H + (1+S)\, d_H e_{\min}\left(\frac{2+K}{3} + q e_B^*\right)\right)\right). \end{aligned} \tag{A81}$$

Thus, (A78) and (A81) define the equilibrium period-1 wages $w_{A1}^*$ and $w_{B1}^*$ that are obtained if we impose a zero profit condition on each worker.

As an alternative, suppose the workers receive the same wages in period 1, based on a zero-profit condition for the incumbent firm as a whole, rather than individual workers. Then each worker receives one half of the sum of wages computed above, i.e., one half of the sum of

(A78) and (A81). This sum of wages is

$$
\begin{aligned}
w_{A1}^* + w_{B1}^* = c_L + \frac{d_L}{2}e_A^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_A^*\right)\right) + (1-\tau)\cdot \\
\left(\frac{1}{2}(1+2K-K^2)\left(c_H + (1+S)\,d_H e_{\min}\left(\frac{2+3K-3K^2+K^3}{3(1+2K-K^2)} + qe_A^*\right)\right)\right. \\
\left. + \frac{1}{2}(1-2K+K^2)\left(c_L + (1+S)\,d_L e_{\min}\left(\frac{1-K}{3} + qe_A^*\right)\right)\right) \\
+ c_L + \frac{d_L}{2}e_B^* - (1-\tau)\left(c_L + d_L e_{\min}\left(\frac{1}{2} + qe_B^*\right)\right) + (1-\tau)\cdot \\
\left(\frac{1}{2}(1+2K-K^2)\left(c_L + (1+S)\,d_L e_{\min}\left(\frac{1+3K-K^3}{3(1+2K-K^2)} + qe_B^*\right)\right)\right. \\
\left. + \frac{1}{2}(1-2K+K^2)\left(c_H + (1+S)\,d_H e_{\min}\left(\frac{2+K}{3} + qe_B^*\right)\right)\right) \\
= (1+\tau)c_L + (1-\tau)c_H + \frac{d_L}{2}(e_A^* + e_B^*) - (1-\tau)d_L e_{\min}(1 + q(e_A^* + e_B^*)) \\
+ (1-\tau)\,(1+S)e_{\min}\left(\frac{1}{2}(1+2K-K^2)d_H\left(\frac{2+3K-3K^2+K^3}{3(1+2K-K^2)} + qe_A^*\right)\right. \\
\left. + \frac{1}{2}(1-2K+K^2)\,d_L\left(\frac{1-K}{3} + qe_A^*\right)\right) \\
+ (1-\tau)\,(1+S)e_{\min}\left(\frac{1}{2}(1+2K-K^2)d_L\left(\frac{1+3K-K^3}{3(1+2K-K^2)} + qe_B^*\right)\right. \\
\left. + \frac{1}{2}(1-2K+K^2)d_H\left(\frac{2+K}{3} + qe_B^*\right)\right).
\end{aligned}
\tag{A82}
$$

This can be simplified to

$$
\begin{aligned}
w_{A1}^{*}+w_{B1}^{*}=&(1+\tau)c_L+(1-\tau)c_H+\frac{d_L}{2}(e_A^{*}+e_B^{*})-(1-\tau)d_L e_{\min}(1+q(e_A^{*}+e_B^{*}))\\
&+(1-\tau)\,(1+S)e_{\min}\frac{1}{2}\Bigg((1+2K-K^2)d_H\left(\frac{2+3K-3K^2+K^3}{3(1+2K-K^2)}\right)\\
&+(1-2K+K^2)d_L\left(\frac{1-K}{3}\right)\\
&+(1+2K-K^2)d_L\left(\frac{1+3K-K^3}{3(1+2K-K^2)}\right)+(1-2K+K^2)d_H\left(\frac{2+K}{3}\right)\\
&+(1+2K-K^2)q(d_H e_A^{*}+d_L e_B^{*})+(1-2K+K^2)q(d_L e_A^{*}+d_H e_B^{*})\Bigg)\\
=&(1+\tau)c_L+(1-\tau)c_H+\frac{d_L}{2}\,(e_A^{*}+e_B^{*})-(1-\tau)d_L e_{\min}(1+q(e_A^{*}+e_B^{*}))\\
&+(1-\tau)\,(1+S)e_{\min}\frac{1}{2}\Bigg(\frac{1}{3}\left(d_L(2+3K^2-2K^3)+d_H(4-3K^2+2K^3)\right)\\
&+(1+2K-K^2)q(d_H e_A^{*}+d_L e_B^{*})+(1-2K+K^2)q(d_L e_A^{*}+d_H e_B^{*})\Bigg).
\end{aligned}
\tag{A83}
$$

Each worker receives one half of this sum, implying that the incumbent makes a profit from one of the workers, and a loss from the other, with zero profit in total.

## A.5 Worker $B$ has a lower effort due to the presence of overconfident worker $A$ (Comparison with a symmetric game)

Consider the benchmark case where the firm hires two "$B$" workers (who are not overconfident), with ability drawn from the Uniform distribution on $[0,1]$, as in the main model.
Denote the *symmetric* equilibrium candidate effort by $\hat{e}_B$. From the main model, recall the probability of winning of a worker of type $B$ as a function of the worker's choice variable, denoted here by $e_B$, evaluated at firms' candidate equilibrium beliefs $\hat{e}_B$ and assuming that the other worker plays the candidate equilibrium effort $\hat{e}_B$.
Recall the marginal promotion probability of worker $B$ in the main model, (A43), and now evaluate at $\hat{e}_B$ for both workers

$$
\begin{aligned}
\left.\frac{\partial P_B}{\partial e_B}\right|_{(\hat{e}_B,\hat{e}_B)} &= \int_{-\infty}^{\infty} f\left(x\right)\frac{x}{\hat{e}_B}f\left(x\right)dx\\
&=\frac{1}{\hat{e}_B}\int_0^1 x dx\\
&=\frac{1}{2\hat{e}_B}.
\end{aligned}
\tag{A84}
$$

Furthermore, recall worker $B$'s first-order condition, (A44), and again evaluate at effort $\hat{e}_B$ for both workers to get

$$\begin{aligned}
c'(\hat{e}_B) &= d_L e_{\min} \left.\frac{\partial P_B}{\partial e_B}\right|_{(\hat{e}_B,\hat{e}_B)} \Big(E[\Theta_A|\Theta_A > \Theta_B] - E[\Theta_A|\Theta_A < \Theta_B]\Big) \\
&= d_L e_{\min} \frac{1}{2\hat{e}_B}\left(\frac{2}{3} - \frac{1}{3}\right) \\
\iff \quad c'(\hat{e}_B)\hat{e}_B &= d_L e_{\min}\frac{1}{6}.
\end{aligned} \tag{A85}$$

The last line of (A85) characterizes the symmetric equilibrium effort that is obtained in a game between two workers in the absence of overconfidence.

Now compare this with worker $B$'s equilibrium first-order condition in the main game, the second line of (A50). As the LHS in both expressions is increasing in effort, the worker has a larger effort in the symmetric game if the two RHS satisfy

$$\begin{aligned}
d_L e_{\min}\frac{1}{6} &> d_L e_{\min}\frac{1}{6}(1-K^2)\frac{1+2K}{1+2K-K^2} \\
\iff 1 &> (1-K^2)\frac{1+2K}{1+2K-K^2}.
\end{aligned} \tag{A86}$$

For our $K \in (0,1)$, we can multiply by $1+2K-K^2 > 0$ and simplify to get $2K^3 > 0$ which is true. Thus, worker $B$ competing with the overconfident worker $A$ in the main game has a lower effort than $B$ would have in a symmetric game with another $B$-worker.

We mention that the second-order condition for the symmetric game holds. The second derivative of the promotion probability is either zero or negative.

Now look at the expected utility of a worker in this symmetric game.

We start from (A54) and evaluate at $\hat{e}_B = e_B = e_B^*$ and $K = 0$, then insert the promotion probability from Lemma 1 (which also holds for $K = 0$) as well as results (c) and (d) of Lemma 2 (which also holds for $K = 0$) to simplify as follows.

$$\begin{aligned}
&\int_{-\infty}^{\infty} F(x)\, f(x)\, dx\; d_L e_{\min}\Big(E[\Theta_A|\Theta_A > \Theta_B] - E[\Theta_A|\Theta_A < \Theta_B]\Big) \\
&\qquad + c_L + d_L e_{\min}\left(E[\Theta_B|\Theta_A > \Theta_B] + q\hat{e}_B\right) - c(\hat{e}_B) \\
&= \frac{1}{2} d_L e_{\min}\left(\frac{2}{3} - \frac{1}{3}\right) + c_L + d_L e_{\min}\left(\frac{1}{3} + q\hat{e}_B\right) - c(\hat{e}_B) \\
&= c_L + d_L e_{\min}\left(\frac{1}{2} + q\hat{e}_B\right) - c(\hat{e}_B).
\end{aligned} \tag{A87}$$

This is the same function of effort as (A90) below ($B$'s payoff in the main game) only with a larger effort.

## A.6 Expected worker payoffs

Here we start from the point in the game where workers choose effort for period 1, and wages for period 1 are sunk. We also ignore the workers' effort cost $c(e_{\min})$ in period 2.

Consider worker $A$, who has a subjective as well as an objective expected payoff. Start with the objective payoff. Recall (A52), but use $f$ instead of $\hat{f}$ and evaluate at $e_A = e_A^*$. Then insert the objective promotion probability from Lemma 1, as well as results (c) and (b) of Lemma 2, to simplify as follows.

$$\begin{aligned}
&\int_{-\infty}^{\infty} F(x+K) f(x)\, dx\, d_L e_{\min}\Big(E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B]\Big) \\
&\qquad + c_L + d_L e_{\min}\left(E[\Theta_A|\Theta_A + K < \Theta_B] + q e_A^*\right) - c(e_A^*) \\
&= \frac{1}{2}\left(1 + 2K - K^2\right)\, d_L e_{\min}\left(\frac{1+2K}{3(1+2K-K^2)}\right) + c_L + d_L e_{\min}\left(\frac{1-K}{3} + q e_A^*\right) - c(e_A^*) \\
&= d_L e_{\min}\left(\frac{1}{6}\frac{(1+2K-K^2)(1+2K)}{(1+2K-K^2)} + \frac{1-K}{3}\right) + c_L + d_L e_{\min} q e_A^* - c(e_A^*) \\
&= c_L + d_L e_{\min}\left(\frac{1}{2} + q e_A^*\right) - c(e_A^*).
\end{aligned} \tag{A88}$$

Now turn to the subjective payoff. Replace the objective by the subjective promotion probability from Lemma 1 above, while the rest remains unchanged, and rewrite as follows.

$$\begin{aligned}
&\int_{-\infty}^{\infty} F(x+K) \hat{f}(x)\, dx\, d_L e_{\min}\Big(E[\Theta_A|\Theta_A + K > \Theta_B] - E[\Theta_A|\Theta_A + K < \Theta_B]\Big) \\
&\qquad + c_L + d_L e_{\min}\left(E[\Theta_A|\Theta_A + K < \Theta_B] + q e_A^*\right) - c(e_A^*) \\
&= \left(1 - \frac{(1-K)^{\gamma+1}}{\gamma+1}\right) d_L e_{\min}\left(\frac{1+2K}{3(1+2K-K^2)}\right) + c_L + d_L e_{\min}\left(\frac{1-K}{3} + q e_A^*\right) - c(e_A^*) \\
&= c_L + d_L e_{\min}\left(\left(1 - \frac{(1-K)^{\gamma+1}}{\gamma+1}\right)\frac{1+2K}{3(1+2K-K^2)} + \frac{1-K}{3} + q e_A^*\right) - c(e_A^*).
\end{aligned} \tag{A89}$$

Obviously, this must be larger than (A88), i.e., the first term in parenthesis is positive. This is because the only difference between (A88) and (A89) is the promotion probability, which is larger in (A89), due to $\hat{f}$ instead of $f$.

For worker $B$ we start from (A54) and evaluate at $e_B = e_B^*$, then insert the promotion probability from Lemma 1 as well as results (c) and (d) of Lemma 2 to simplify as follows.

$$\begin{aligned}
&\int_{-\infty}^{\infty} F\left(x-K\right) f\left(x\right) dx\, d_L e_{\min}\Big(E[\Theta_A|\Theta_A+K>\Theta_B]-E[\Theta_A|\Theta_A+K<\Theta_B]\Big)\\
&\qquad + c_L + d_L e_{\min}\left(E[\Theta_B|\Theta_A+K>\Theta_B]+qe_B^*\right)-c(e_B^*)\\
&=\frac{1}{2}\left(1-2K+K^2\right)\, d_L e_{\min}\left(\frac{1+2K}{3(1+2K-K^2)}\right)+c_L+d_L e_{\min}\left(\frac{1+3K-K^3}{3(1+2K-K^2)}+qe_B^*\right)-c(e_B^*)\\
&=c_L+d_L e_{\min}\frac{1}{6}\left(\frac{(1-2K+K^2)(1+2K)+2(1+3K-K^3)}{(1+2K-K^2)}\right)+d_L e_{\min} q e_B^*-c(e_B^*)\\
&=c_L+d_L e_{\min}\frac{1}{6}\left(\frac{3(1+2K-K^2)}{(1+2K-K^2)}\right)+d_L e_{\min} q e_B^*-c(e_B^*)\\
&=c_L+d_L e_{\min}\left(\frac{1}{2}+qe_B^*\right)-c(e_B^*).
\end{aligned} \tag{A90}$$

Summarizing, the objective expected payoffs are

$$\begin{aligned}
&c_L+d_L e_{\min}\left(\frac{1}{2}+qe_A^*\right)-c(e_A^*)\\
&c_L+d_L e_{\min}\left(\frac{1}{2}+qe_B^*\right)-c(e_B^*).
\end{aligned} \tag{A91}$$

Comparing these two expressions, it is clearly conceivable that worker $B$'s payoff can be higher or lower than worker $A$'s. Numerical examples confirm that both situations are consistent with equilibrium.

## A.7 Proof of Proposition 2

The following lemma derives a result we use in the proof of the proposition.

**Lemma 6.** *For a constant $K \in (-1, 0)$, and $f$ and $\hat{f}$ defined in (A1) and (A2), we have*

$$E\left[\Theta_A|\,\Theta_A+K>\Theta_B\right]-E\left[\Theta_A|\,\Theta_A+K<\Theta_B\right]=\frac{1-2K}{3(1-2K-K^2)}. \tag{A92}$$

*Proof of Lemma 6.* Define $K' := -K$, where $K' \in (0, 1)$. We observe

$$\begin{aligned}
&E\left[\Theta_A|\,\Theta_A+K>\Theta_B\right]\\
=\;&E\left[\Theta_A|\,\Theta_A-K'>\Theta_B\right]\\
=\;&E\left[\Theta_A|\,\Theta_A>\Theta_B+K'\right].
\end{aligned}$$

Since $\Theta_A$ and $\Theta_B$ are iid, this is the same expectation as in Lemma 2, part (e). Hence,

$$E\left[\Theta_A|\,\Theta_A>\Theta_B+K'\right]=\frac{2+K'}{3}=\frac{2-K}{3}.$$

Furthermore,

$$\begin{aligned} & E\left[\Theta_A \middle| \Theta_A + K < \Theta_B\right] \\ = \; & E\left[\Theta_A \middle| \Theta_A - K' < \Theta_B\right] \\ = \; & E\left[\Theta_A \middle| \Theta_A < \Theta_B + K'\right]. \end{aligned}$$

Since $\Theta_A$ and $\Theta_B$ are iid, this is the same expectation as in Lemma 2, part (d). Hence,

$$\begin{aligned} E\left[\Theta_A \middle| \Theta_A < \Theta_B + K'\right] &= \frac{1 + 3K' - (K')^3}{3\left(1 + 2K' - (K')^2\right)} \\ &= \frac{1 - 3K + K^3}{3\left(1 - 2K - K^2\right)}. \end{aligned}$$

We obtain

$$\begin{aligned} & E\left[\Theta_A \middle| \Theta_A + K > \Theta_B\right] - E\left[\Theta_A \middle| \Theta_A + K < \Theta_B\right] \\ = \; & \frac{2-K}{3} - \frac{1 - 3K + K^3}{3\left(1 - 2K - K^2\right)} \\ = \; & \frac{2 - 4K - 2K^2 - K + 2K^2 + K^3 - 1 + 3K - K^3}{3\left(1 - 2K - K^2\right)} \\ = \; & \frac{1 - 2K}{3\left(1 - 2K - K^2\right)}. \end{aligned}$$

□

*Proof of Proposition 2.* Suppose there is an upper limit for effort, $\bar{e} > 0$. This limit is set equal to the symmetric equilibrium effort $\hat{e}$ that is obtained in the absence of overconfidence, i.e., in a game between two workers of type $B$. This effort is implicitly given by $c'(\hat{e})\,\hat{e} = \frac{d_L e_{\min}}{6}$ and has been derived in Section A.5, see (A85). We wish to show that there is a Nash equilibrium of this modified game in which both workers choose effort $\bar{e} = \hat{e}$, i.e., $e_A = e_B = \hat{e}$.

The analysis proceeds in a way similar to that of the main game. In particular, the incumbent and outside firms have given beliefs which are confirmed in equilibrium, and they update their beliefs in the same way as before.

First, consider worker $B$. By the analysis in Section A.5, in particular, (A85), it is clear that effort $\hat{e}$ defined by $c'(\hat{e})\,\hat{e} = \frac{d_L e_{\min}}{6}$ satisfies $B$'s first-order condition, when playing against this effort. Thus, $\hat{e}$ is a best response for $B$ when $A$ plays $\hat{e}$.

Second, consider worker $A$. Proceeding by way of contradiction, we assume that worker $A$ optimally responds with $e_A < \hat{e}$ to $e_B = \hat{e}$, and the market (i.e., the firms) correctly anticipates this response when determining wage offers. This implies $K < 0$. Similar to the main model, we assume cost functions such that, in equilibrium, both workers always have positive promotion probability, i.e., $|q(e_A - e_B)| < 1$, such that $K < 0$ implies $K \in (-1, 0)$.

Recall $A$'s marginal promotion probability (A37), which we now evaluate at the candidate

$(e_A, e_B)$ with $\tilde{e}_A = e_A < e_B = \hat{e} = \tilde{e}_B$ and $K = q(e_A - \hat{e})$. We use Lemma 3(a), case $-1 < K \leq 0$, to get

$$\left.\frac{\partial \hat{P}_A}{\partial e_A}\right|_{(e_A,e_B)} = \int_{-\infty}^{\infty} f(x+K)\frac{x}{e_A}\hat{f}(x)\,dx = \frac{1}{e_A}\frac{\gamma}{1+\gamma}\left(1+K(-K)^{\gamma}\right). \tag{A93}$$

Recalling (A40) and (A41), $A$'s marginal utility can be written as

$$\frac{d_L e_{\min}}{e_A}\frac{\gamma}{1+\gamma}\left(1+K(-K)^{\gamma}\right)\Delta\Theta - c'(e_A), \tag{A94}$$

where $\Delta\Theta := (E[\Theta_A|\,\Theta_A + K > \Theta_B] - E[\Theta_A|\,\Theta_A + K < \Theta_B])$.
Suppose that $\frac{\gamma}{1+\gamma}\left(1+K(-K)^{\gamma}\right)\Delta\Theta \geq \frac{1}{6}$. Then $\frac{d_L e_{\min}}{e_A}\frac{\gamma}{1+\gamma}\left(1+K(-K)^{\gamma}\right)\Delta\Theta - c'(e_A) > \frac{d_L e_{\min}}{6\hat{e}} - c'(\hat{e}) = 0$. Recall that the equality holds by the definition of $\hat{e}$, see (A85). This would mean that $A$ always wishes to deviate to a higher effort and give us the desired contradiction.
It remains to show that

$$\frac{\gamma}{1+\gamma}\left(1+K(-K)^{\gamma}\right)\Delta\Theta \geq \frac{1}{6}$$

for all (relevant) $K \in (-1, 0)$ and $\gamma > 1$. By Lemma 6, this is equivalent to

$$\frac{\gamma}{1+\gamma}\left(1+K(-K)^{\gamma}\right)\frac{1-2K}{3(1-2K-K^2)} \geq \frac{1}{6}.$$

Note that $\frac{\gamma}{3(1+\gamma)} \geq \frac{1}{6}$. Hence, the condition would be fulfilled if

$$\begin{aligned}
& \left(1+K(-K)^{\gamma}\right)\frac{1-2K}{1-2K-K^2} \geq 1 \\
\Leftrightarrow\quad & \left(1+K(-K)^{\gamma}\right)(1-2K) \geq 1-2K-K^2 \\
\Leftrightarrow\quad & 1-2K+K(-K)^{\gamma}-2K^2(-K)^{\gamma} \geq 1-2K-K^2 \\
\Leftrightarrow\quad & K(-K)^{\gamma}-2K^2(-K)^{\gamma} \geq -K^2 \\
\Leftrightarrow\quad & (-K)^{\gamma}-2K(-K)^{\gamma} \leq -K \\
\Leftrightarrow\quad & (-K)^{\gamma-1}+2(-K)^{\gamma} \leq 1.
\end{aligned}$$

Since $\gamma - 1 > 0$ and $-K \in (0, 1)$, we have $(-K)^{\gamma-1} < 1$. Moreover, since $\gamma > 1$, we have $\lim_{K\to 0} 2(-K)^{\gamma} = 0$. By the continuity of $(-K)^{\gamma-1} + 2(-K)^{\gamma}$, it then follows that the condition $(-K)^{\gamma-1} + 2(-K)^{\gamma} \leq 1$ is always fulfilled for $K$ sufficiently close to zero. Denote by $\left[-\bar{K}, 0\right)$ the set of values for $K$ for which the condition is met (with $\bar{K} > 0$).
For a given $q > 0$, by assuming sufficiently steep cost functions, we can always ensure that equilibrium efforts $e_A$ and $e_B$ satisfy $e_A, e_B \leq \frac{\bar{K}}{q}$. In this case, it follows that $K \in \left[-\bar{K}, \bar{K}\right]$ so that we can restrict attention to $K \in \left[-\bar{K}, 0\right)$ when considering $e_A < e_B$.

□